Results in Physics 10 (2018) 411–423

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

An acoustic modeling of the three-dimensional annular segment cavity with various impedance boundary conditions

Yujie Xiao[a], Dong Shao[a,b], Hong Zhang[c], Cijun Shuai[d], Qingshan Wang[d,*]

[a] Naval Research Academy, Beijing 100161, PR China
[b] Beijing Aerospace Technology Institute, China Aerospace Science & Industry Corp., Beijing 100074, PR China
[c] College of Mechanical and Electrical Engineering, Harbin Engineering University, Harbin 150001, PR China
[d] State Key Laboratory of High Performance Complex Manufacturing, Central South University, Changsha 410083, PR China

ARTICLE INFO

Keywords:
Fourier series
Annular segment cavity
Impedance wall
Rayleigh-Ritz technique

ABSTRACT

A three-dimensional Fourier series method (3D-FSM) is applied to study the acoustic characteristics of annular segment acoustic cavity with various impedance boundary conditions. The formulation is constructed to describe the cavity system based on the energy principle. Under the framework of this paper, the admissible sound pressure function is generally set, regardless of boundary conditions, to a 3D Fourier cosine series and six supplementary functions. These supplementary functions can eliminate the discontinuous or jumping phenomenon in the boundaries. All the series expansion coefficients can be obtained through the Rayleigh-Ritz technique. The results obtained by the present method in this paper show good convergence. The accuracy of the present method is verified by being compared with the exact solution and the finite element method (FEM). The natural frequencies and modal shapes of the annular segment cavity are studied. The sound pressure response is investigated under the excitation of a monopole source inside the cavity. In this paper, some results of the acoustic characteristics of the cavity with various geometric parameters and boundary conditions are obtained, such as angle, radius ratio, impedance value and the number of impedance wall. These results provide a benchmark for the future researches.

Introduction

As the basic structure, the annular segment cavity is widely used in engineering application, such as aerospace, marine engineering, civil construction, rockets and rail transportation. Therefore, there is important applicative value for the study on annular segment cavity system and cylindrical cavity system which is of great significance in the acoustic design and noise control of complex acoustic space. So far, the study on acoustic properties of enclosed space has been well-rounded. These results provide an important theoretical basis for the further research. But, there are some limitations in the study on the varying impedance boundary conditions. Therefore, this paper aims to study the acoustic characteristics of annular segment cavity with various impedance boundary conditions.

The rectangular cavities, as the simplified model of room acoustics, have been extensively investigated. There have been many research results on acoustic characteristics of the two-dimensional (2D) rectangular cavities with rigid-wall and impedance-wall boundary conditions, which have built a good fundament for the future research. Koch [1] computed numerically in 2D rectangular deep and shallow open cavity. Huang and Jiang [2] discussed the circular line sound source model for 2D thin acoustic cavity by using the transfer matrix method. This model avoided the singularity of source and obtained the uniform pressure responses on the circular line. Dhandole and Modak [3] extended the sequential quadratic programming algorithm to solve the constrained optimization problem, which is used to update the acoustic finite element model. The sound pressure responses of a 2D rectangular cavity and a car cavity were verified based on this method. González et al. [4] used spectral element spatial discretization to analyze the acoustic resonances of a 2D open cavity with absorbing boundary conditions by solving a multi-dimensional Helmholtz equation. Aktas et al. [5] considered the Navier-Stokes equations to simulate a 2D rectangular enclosure filled by compressible gas. The frequencies were considered along the enclosure in which the oscillatory flow field is created through the vibration of the left wall of the enclosure.

With the gradual deepening of the research, the study on the acoustic characteristics of the 3D rectangular cavity or rectangular-like cavity model has attracted wide attention. Huang and Jiang [6] extended Ewald's summation technique to calculate acoustic Green's function for closed rectangular cavity. This transformed Green's

* Corresponding author.
E-mail address: qingshanwang@csu.edu.cn (Q. Wang).

https://doi.org/10.1016/j.rinp.2018.06.039
Received 25 April 2018; Received in revised form 30 May 2018; Accepted 18 June 2018
Available online 22 June 2018

function showed fast convergence. Larbi et al. [7] applied a new finite element formulation to study the internal acoustic problems with absorbing walls by introducing the normal fluid displacement field on the damped walls. Pàmies et al. [8] investigated the sound radiation from an aperture in a rectangular enclosure under low modal conditions by using the Rayleigh radiation equation. Li and Cheng [9,10] investigated acoustic modes of a rectangular-like cavity with a slight geometrical distortion introduced by a leaning wall. Petyt et al. [11] developed an isoparametric acoustic finite element model with twenty nodes, which was first used to study the rectangular enclosure. Petyt et al. [12] obtained some results of the rectangular cavity containing a rigid and incomplete partition by using FEM. Pan et al. [13] introduced the modal expansion approach to study the sound pressure response of rectangular enclosures which had suitable modification of the damping and frequency shift on the rigid-wall. The Helmholtz resonance effects of the cockpit in a helicopter were also considered in the model. Nabavi et al. [14] reported an experimental analysis of the non-linear phenomena of regular and irregular acoustic streaming patterns which were in a square air-filled channel with rigid-wall based on the synchronized particle image velocimetry technique. Guha et al. [15] presented a partly flexible laminated composite enclosure based on finite element free vibration analysis and boundary element solver. Du et al. [16] studied the acoustic characteristics of the rectangular cavity with general impedance boundary conditions. A variety of boundary conditions of rectangular cavity are studied, including rigid wall, impedance wall and fully absorbing wall. A lot of existing researches are made on the rectangular and rectangular-like cavity, but the studies on other shape cavities are rare. Shi et al. [17] proposed an analytical to study the vibro-acoustic behaviors of a double-plate structure with an acoustic cavity. In this model, two dimensional (2D) and three dimensional (3D) modified Fourier series were used to represent the displacement of the panels and the sound pressure inside the cavity. Applying the modified variational method, Xie et al. [18,19] studied the acoustic properties and vibro-acoustic responses of irregular enclosures. The Chebyshev polynomial exhibits high performance at entire field approximations. Bouillard et al. [20] developed an improved element-free Galerkin method for the acoustic problem of three-dimensional complex geometries. Bouzouane et al. [21] investigated the ultra-thin films effects on vibro-acoustic behavior of a laminated glass plate composed of two elastic skins, a viscoelastic core and two ultra-thin adhesive films based on the classical plate's theories. Following the energy method, Zhang et al. established a composite thin plate-cavity coupling system [22] based on CPT and the moderately thick laminated rectangular plate-cavity coupling system [23] based on FSDT. Shi et al. [24] presented a general solution method to predict the dynamic behaviors of the three-dimensional (3D) acoustic coupled system of a partially opened cavity coupled with a flexible plate and an exterior field of semi-infinite size. Later, Shi et al. [25] extended this solution to further study the modeling and acoustic eigenanalysis of coupled spaces with a coupling aperture of variable size.

In real-world applications, there are many cylindrical acoustic cavities, such as rocket, submarine, aircraft cabin and so on. The existing literature researches for rotary-shaped annular segment cavities and cylindrical cavities have obvious limitations. Laulagnet et al. [26] used modal analysis to study acoustic radiation of finite cylindrical shell which was immersed in heavy fluid. In this paper, the related concepts of radiation and damped modes were introduced. In addition, this study also gave an important discovery that the behavior of a shell in water is very different from that of one in air. Gardonio et al. [27] investigated the plane wave transmission characteristic of a circular cylindrical sandwich-shell based on the modal interaction analysis. The model established in this paper can be used to study the high order acoustic modes and investigate the sensitivity to damp and cavity absorption. Yang et al. [28] used an integro-modal approach to deal with sound radiation from a finite cylindrical shell with an irregular-shaped acoustic enclosure. The cylindrical shell contains a machinery equipment modelled as a rectangular object attached to shell with a spring-mass system. In addition, effects of the object size on the coupling between acoustic modes and structural modes were investigated. Shi et al. [29] proposed the acoustic radiation force function on a solid elastic spherical particle placed in an infinite rigid cylindrical cavity filled with an ideal fluid. It is not difficult to find that there are few studies on the annular segment cavity or cylindrical cavity with various impedance boundary conditions.

Considering the restrictions of the rotary-shaped cavity with various impedance walls in the current researches, the acoustic characteristic analyses of the annular segment cavity with rigid-wall and various impedance-wall boundary conditions are developed. Based on the Rayleigh-Ritz energy technique, a 3D Fourier series solution is extended to study the sound properties of annular segment cavity with various impedance boundary conditions. This method is previously proposed by Du et al. [16] to study the rectangular cavity with general impedance boundary conditions. The sound pressure functions can be written as a feasible superposition of the 3D trigonometric series expansion, ignoring the effect of boundary conditions, in spectral form, as a 3D Fourier cosine series and six supplementary functions. On the basis of traditional Fourier series, these supplementary functions are added to eliminate the discontinuous or jumping phenomenon in the boundaries which are regarded as a periodic function and defined within the entire coordinates of the cavity. All these unknown coefficients are defined in the generalized coordinates which can be solved by Rayleigh-Ritz procedure. In our previous work [30–41], this kind of Fourier series method has been used to investigate the vibration characteristics of 2D solid structures. The results obtained by the present method are compared with those results obtained by literatures and FEM, which shows good agreement. The current work mainly deals with the natural frequencies, mode shapes, and sound pressure responses with significant constraints such as arbitrary sector angle, geometric dimension and various values of impedance parameters. These parametric studies have guiding significance for the acoustic noise control of annular segment cavity.

Theoretical formulations

Description of the model

As shown in Fig. 1, an annular segment cavity model is established to analyze characteristics of the three-dimensional sound field. For the annular segment cavity, the dimensions of the cavity with general wall impedance are listed here: outer radius, inner radius, height and sector angle dimensions are R_1, R_2, h and ϕ, respectively. An integral cylindrical coordinate system (O-$r\,\theta\,z$) and a local-coordinate system (O'-s $\theta\,z$) in the cavity model are established. In local-coordinate system, s represents the difference between the outer radius and the inner radius which is expressed as R_1-R_2. Besides, θ and z represent the angle and height directions of the studied model. A monopole source Q is located at a corner of the annular segment cavity.

Admissible sound pressure functions

The modal characteristics of annular segment closed space under various impedance boundaries can be obtained by solving 3D homogeneous Helmholtz acoustic equation and the boundary value problem of the impedance acoustic boundary constraint equation. It is assumed that the distribution of impedance on the wall is same and the attenuation effect of medium viscosity is ignored in sound propagation process. Under this condition, the boundary value problem can be described as:

$$\frac{\partial p}{\partial n} = -j\frac{\rho c_0}{Z_i}kp \tag{1}$$

where p, c_0 and ρ are the acoustic pressure, the speed of sound