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Fluctuations in strongly correlated electron systems

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ABSTRACT

High transition temperature superconductors in cuprates exhibit the charge-density-wave fluctuations and the ferromagnetic time-reversal-symmetry-breaking fluctuation in the polar Kerr rotation experiments. We demonstrate that they share the same root of origin, and the underlying mechanism also leads to the pseudogap formation. The pseudogap formation, the charge-density-wave fluctuation, and the time-reversal-symmetry-breaking fluctuation are the consequent phenomena of the correlation. They are the basic notions in strongly correlated electron systems.

Introduction

Correlated electrons have exhibited many interesting phenomena that deviate from the Fermi liquid theory and the theory of phase transition. Taking the high transition temperature superconductors in cuprates as an example [1], the pseudogap formation [2], charge-density-wave fluctuations observed in the scanning tunneling microscopes and resonant soft X-ray scattering experiments [3–6], and ferromagnetic time-reversal-symmetry-breaking fluctuations [7], occur simultaneously wide in the phase diagram. The onset of the time-reversal-symmetry-breaking fluctuation coincides with formation temperature of the pseudogap [7]. The charge-density-wave fluctuation resides well in the pseudogap phase [8]. Those fluctuations have one common property. Namely, there are no signatures of phase transition as they occur. Their origins are mysterious. In this paper, we will demonstrate that charge-density-wave fluctuation and time-reversal-symmetry-breaking fluctuation are actually universal, if the correlated electrons have the pseudogap phase.

Recently, one of us (CHC) proposed a theory of the pseudogap formation [9]. The electrons *weakly* interacting with the U(1) gauge field, originated from the spin Berry's phase [10], open a gap-like structure, when the gauge field acquires the mass. The mass acquisition of the gauge field is due to the *strong coupling* with the anti-ferromagnetic *fluctuation*, a remnant of the anti-ferromagnetism as the system is doped. The basic assumption of this theory is that the spin anisotropy is a relevant perturbation, so that the anti-ferromagnetic fluctuation can be described by a phase field, $\phi(\vec{x}, t) = \frac{1}{q} e^{i\sigma(\vec{x}, t)}$, where q is the coupling between the gauge field and the anti-ferromagnetic fluctuation. We emphasize that the anti-ferromagnetic fluctuation does not couple to the elections directly. In two dimensions, the Kosterlitz-Thouless (KT) transition takes place for the phase field at finite temperature. Then, the anti-ferromagnetic fluctuation is absorbed by the gauge transformation and becomes the *longitudinal* component of the gauge field. Because the gauge field acquires mass, the interaction between electrons becomes short-ranged. Due to the nature of the KT transition, there are no conventional signatures of phase transitions. Translational symmetry and the time reversal symmetry are well preserved.

At the first glance, it looks contradictory that the phase, preserving both the translational and time reversal symmetries, hosts the charge-density-wave fluctuation and the time-reversal-symmetry-breaking fluctuation. We will show later that they are fluctuations and not the ordering states. Electronic interaction mediated by the gauge field infers that electrons exchange virtual particles of the pure imaginary wave vectors. Nonetheless, due to the quantum fluctuation, gauge field can be excited in the propagation modes of the real wave vector. The charge-density-wave fluctuation is the direct consequence of the propagating *gauge-electric field* contributed from the *longitudinal* mode. On the other hand, the ferromagnetic time-reversal symmetry-breaking fluctuation originates from the propagating *gauge-magnetic field* of the transverse modes.

This paper is organized as the following. We will discuss the effective interaction between electrons by integrating out the electronic degree of freedom. The propagation modes of the gauge field can be obtained by solving the classical equations of motion. Then, we consider the classical motion of the electrons in the presence of the propagating gauge field. We will apply the current scheme to the high-T_c superconductors. The presence of the anti-ferromagnetic fluctuation and the emergence of the gauge interaction baptize the quantum correlation. Once it is considered carefully, many of pseudogap phenomenology can be realized. The pseudogap formation, the charge-density-

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wave fluctuation, and the time-reversal-symmetry-breaking fluctuation do not have the relation of causality. They are all the consequent phenomena of the correlation.

Fluctuations

Effective theory of the gauge field

As the cuprates are doped, the anti-ferromagnetic ordering ceases, the pseudogap phase is developed, and the gapless states are generated in the nodal directions. It turns out that pseudogap structure is anisotropic in the momentum space, which we believe that it is the sum of the two causes: one mechanism to open an isotropic gap [9] and the another mechanism to introduce the nodal quasiparticles [11]. In this paper, we do not explain the phenomena associated with the nodal quasiparticles. We focus on the consequences that relate to the pseudogap. Let us consider the following Lagrangian density [9]

$$
\begin{aligned}
\mathscr{L} &= \mathscr{L}_\psi + \mathscr{L}_a + \mathscr{L}_\phi, \\
\mathscr{L}_\psi &= \sum_\alpha \psi_\alpha^\dagger (i\partial_0)\psi_\alpha - \tfrac{1}{2m}\left[\left(-\tfrac{\vec{\nabla}}{i} - g\vec{a}\right)\psi_\alpha^\dagger\right]\left[\left(\tfrac{\vec{\nabla}}{i} - g\vec{a}\right)\psi_\alpha\right] - g a_0 \psi_\alpha^\dagger \psi_\alpha, \\
\mathscr{L}_a &= -\tfrac{1}{4} f_{\mu\nu} f_{\mu\nu}, \\
\mathscr{L}_\phi &= \tfrac{1}{2} M_0^2 (D_0\phi)^\dagger (D_0\phi) - \tfrac{1}{2} M_1^2 (D_i\phi)^\dagger (D_i\phi),
\end{aligned} \tag{1}
$$

where ψ_α is the electron variable with the spin index α, $(a_0, \vec{a})$ is the gauge field, g is the coupling of the electrons to the gauge field, M_0 and M_1 are the mass parameters, and $D_0 = i\partial_0 - q a_0$ and $D_i = -i\partial_i - q a_i$ are the covariant derivative. In Eq. (1), we adopted the natural unit, where $\hbar$ and the speed of light c are set to be 1. It is equivalent to roughly set 197 eV· nm = 1, which indicates that the mass of the gauge field defines the length scale. In cuprates, the wavelength of the charge density wave appears to be the only length scale, which implies $M_0 = M_1$. Considering together the pseudogap magnitude, about 40 meV [12], the dimensionless gauge coupling $\frac{g^2}{2m}$ can be computed $\sim 1.5 \times 10^{-3}$ [9]. The weak-coupling nature allows us to compute the effective Lagrangian of the gauge field and the ϕ field perturbatively. Integrating out the electronic degrees of freedom, the diagrams that renormalize the gauge coupling are given in Fig. 1. Using the Green's function of electrons for the insulators [13,14], those diagrams are zero. Namely, the gauge coupling is not renormalized by the electrons.

This result implies that the pseudogap magnitude and the onset temperature are independent of the external magnetic field [15–19]. It is because the external magnetic field couples only to the electrons, and they have no contribution to renormalize the gauge coupling and the mass of the gauge field.

Having integrated out the electron degrees of freedom, the classical equations of motion of the gauge field and the $\phi = \frac{1}{q}e^{i\sigma}$ field can be derived.

$$
\begin{aligned}
&\vec{\nabla}\cdot(\vec{\nabla} a_0 + \partial_t \vec{a}) = M_0^2\left(\tfrac{1}{q}\partial_t\sigma + a_0\right) \\
&\vec{\nabla}\times(\vec{\nabla}\times\vec{a}) = M_1^2\left(\tfrac{1}{q}\vec{\nabla}\sigma - \vec{a}\right) - \partial_t(\vec{\nabla} a_0 + \partial_t\vec{a}) \\
&M_0^2\partial_t\left(\tfrac{1}{q}\partial_t\sigma + a_0\right) = M_1^2\vec{\nabla}\cdot\left(\tfrac{1}{q}\vec{\nabla}\sigma - \vec{a}\right).
\end{aligned} \tag{2}
$$

The Hamiltonian density can be also computed.

$$
\mathscr{H} = \frac{1}{2}(E^2 + B^2) + \left(\frac{1}{q}\partial_t\sigma + a_0\right)\vec{\nabla}\cdot(\vec{\nabla} a_0 + \partial_t\vec{a}) - \frac{M_0^2}{2}\left(\frac{1}{q}\partial_t\sigma + a_0\right)^2 + \frac{M_1^2}{2}\left(\frac{1}{q}\vec{\nabla}\sigma - \vec{a}\right)^2, \tag{3}
$$

where $\vec{E} = -\vec{\nabla} a_0 - \partial_t\vec{a}$ is the gauge-electric field and $\vec{B} = \vec{\nabla}\times\vec{a}$ is the gauge-magnetic field. Solving Eq. (2) in the pseudogap phase, where the expectation value of σ vanishes, we obtain

$$
\begin{aligned}
&\left(\partial_t^2 - \tfrac{M_1^2}{M_0^2}\nabla^2 + M_1^2\right)a_0 = 0 \\
&\left(\partial_t^2 - \nabla^2 + M_1^2\right)\vec{a} = \left(\tfrac{M_0^2}{M_1^2} - 1\right)\partial_t(\vec{\nabla} a_0)
\end{aligned} \tag{4}
$$

There are two solutions in Eq. (4). The longitudinal mode has the dispersion relation $\omega_L^2 = \frac{M_1^2}{M_0^2}k_L^2 + M_1^2$, and the transverse mode has the dispersion relation $\omega_T^2 = k_T^2 + M_1^2$.

Fluctuation of the density modulation

In the high temperature phase, the $\phi(\vec{x}, t)$ field is fluctuating. The gauge field is massless containing only the transverse mode. In the pseudogap phase, the $\phi(\vec{x}, t)$ field picks up a quasi-long-ranged order through the Kosterlitz-Thouless transition and becomes the longitudinal mode of the gauge field via the gauge transformation [9]. Interestingly, the longitudinal mode has only $\vec{E}$ field and no $\vec{B}$ field. Excited by the quantum fluctuation, the $\vec{E}$ field gives the non-trivial dynamics to the electrons. Without losing generality, we consider the standing-wave solution and take the x direction as the longitudinal direction, $a_0 = A_0 e^{ik_L x}\cos(\omega_L t)$ and $\vec{a} = \frac{-ik_L\omega_L}{\omega_L^2 - M_1^2}A_0 e^{ik_L x}\sin(\omega_L t)\hat{x}$, where A_0 is the strength of the quantum fluctuation, and its magnitude will be determined shortly. The energy density of the longitudinal mode can be computed $\mathscr{E}_L = \frac{A_0^2}{4}\left(\frac{M_0^4}{k_L^2} + M_0^2\right)$. Likewise, the energy density of the transverse mode can be computed $\mathscr{E}_T = \frac{A_1^2}{4}(k_T^2 + M_1^2)$, if $\vec{a} = A_1 e^{ik_T x}\cos(\omega_T t)\hat{y}$, where A_1 is the strength of the quantum fluctuation.

Apparently, the longitudinal mode and the transverse mode have very different characters. From their energy density, the longitudinal mode favors a big k_L, and the transverse mode favors a long wavelength k_T. Therefore, the $\vec{E}$ field modulation of the longitudinal mode must be in the lattice scale, and the $\vec{B}$ field of the transverse mode favors the uniform distribution. As we will see later, the former is the driving force of the charge-density-wave fluctuation. Driven by the longitudinal mode, the electrons acquire the kinetic energy to form the orbital

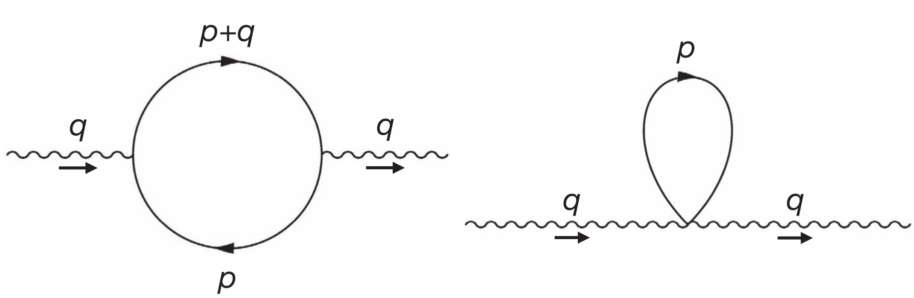

Fig. 1. Feynman diagrams to compute the effective Lagrangian of the gauge field, that is proportional to $f_{\mu\nu}f_{\mu\nu}$ in the long wavelength limit.