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Controlled ultrafast transfer and stability degree of generalized coherent states of a kicked two-level ion

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We investigate quantum dynamics of a two-level ion trapped in the Lamb-Dicke regime of a δ-kicked optical lattice, based on the exact generalized coherent states rotated by a $\pi/2$ pulse of Ramsey type experiment. The spatiotemporal evolutions of the spin-motion entangled states in different parameter regions are illustrated, and the parameter regions of different degrees of quantum stability described by the quantum fidelity are found. Time evolutions of the probability for the ion being in different pseudospin states reveal that the ultrafast entanglement generation and population transfers of the system can be analytically controlled by managing the laser pulses. The probability in an initially disentangled state shows periodic collapses (entanglement) and revivals (de-entanglement). Reduction of the stability degree results in enlarging the period of de-entanglement, while the instability and potential chaos will cause the sustained entanglement. The results could be justified experimentally in the existing setups and may be useful in engineering quantum dynamics for quantum information processing.

Introduction

Individual trapped ions constitute one of the most promising systems for implementing quantum information processing [1–4] and quantum computing [5–9]. Quantum logic operations, which require two essential ingredients: a stable particle system and high fidelity quantum states [10,11], are crucial building blocks in any scheme of universal quantum computing. For developing quantum technologies based on trapped ion systems, one of the major difficulties is the suppression of the instability-induced decoherence in laser-ion interactions [12–15]. Stabilization of a single ion in Lamb-Dicke regime is an important base for applications based on trapped single ions [16]. Most of the well-developed methods for the stability analysis of classical systems cannot be directly applied to quantum systems due to the different kinds of specifically quantum dynamical instabilities differing from classical instabilities [12–14,17–19]. Therefore, quantum fidelity was first proposed by Peres as a measurement of the quantum stability defined as the overlap between a considered quantum state and its initial state [20]. High-fidelity state preparation has been demonstrated in experiments of trapped ions providing a vital support for quantum information processing [10]. We have known that the generalized coherent states of a harmonically trapped ion can show good correspondence between classical and quantum motions [21–23], and the manipulation of the coherent-state wave packets of two trapped ions performs a fundamental two-qubit gate which provides the basic ingredient for quantum computation [24]. So the stability of exact generalized coherent states is worthy of further investigation.

Many efforts have been put into the designing laser fields for reaching a selected state of a quantum system [25,26], and different laser fields are applied with the aim to control the quantum dynamics of trapped ions or neutral atoms [27,28]. Especially periodic arrays of ultrashort laser pulses which are theoretically approximate to delta pulses have generated rich physical phenomena [29–33]. The interesting ultrafast entanglement of a single atomic hyperfine spin state with its motional state has been demonstrated experimentally when the atom is exposed to a short train of picosecond laser pulses [30,33]. For a delta-kicked two-level ion, the singularity of the laser-ion kicked interaction may cause the phase hops of the motional states [34]. The detectable probabilities of different pseudospin states hop at the kick moments because of the jumping phases of the motional states [12,35]. Using the Floquet time evolution operators, the quantum state just after (or before) a kick is given theoretically [12,30,35–37]. By contrast, we have given a method to construct a set of exact solutions of the

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generalized coherent states which can render more transparent strategies to control the system [18,23,38].

In this paper, we consider a single two-level ion trapped in the Lamb-Dicke regime of a δ-kicked optical lattice. Applying the exact solutions of generalized coherent states rotated by a $\pi/2$ pulse [12,38] and considering the stability region of parameter space where classical stability criterion fully agrees with fidelity treatment of quantum ground-state stability, we go further to investigate instability degree and the ultrafast population transfers of the system. For different system parameters, we illustrate the spatiotemporal evolutions of the spin-motion entangled states and time evolutions of the ground-state fidelity. A stability criterion that we redefine provides an approach to experimentally control the instability degree of the ion by adjusting the system parameters. Meanwhile, the probability $P_\alpha(t)$ for the ion being in the internal pseudospin state $|g_\alpha\rangle$ is obtained as a function of time, so the time evolution of the probability within a laser-kicked period can be illustrated exactly. At the first kick moment, the probability changes rapidly from $P_\alpha(T-)$ to $P_\alpha(T+)$ with $T+(T-)$ the time just after (before) a kick, which is able to demonstrate ultrafast population transfer and entanglement generation. Finally, we explore the link between the probability $P_\alpha(t)$ and the different degrees of stability. The probability in an initially disentangled state exhibits regular dynamics in the form of collapses and revivals for the parameters in the stability region, and decreasing the stability degree can result in enlarging the revival period, which means a longer period of de-entanglement. While in the instability region, the probability aperiodically oscillates and collapses to $\frac{1}{2}$ with increasing the time, which implies the sustained entanglement and a hidden partnership between the quantum entanglement and the potential chaos [29]. These results may be useful in engineering quantum dynamics for quantum information processing.

Rotated generalized coherent states and their stability degrees

We consider a single trapped two-level ion with two stable internal pseudospin states $|g_\alpha\rangle$ for $\alpha = 1, 2$, which is confined in a δ-kicked optical lattice. The quantum dynamics of the system is governed by the Hamiltonian [12]

$$H(x,t) = H_0 + \sum_{\alpha=1}^{2} V_\alpha \cos(2kx)|g_\alpha\rangle\langle g_\alpha|\sum_{j=1}^{\infty}\delta(t-jT), \tag{1}$$

where $H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$ is the harmonic Hamiltonian, m and ω are the ion mass and trap frequency, k and T denote the laser wavevector and kicked period, and V_α is the laser-ion interacting strength which is related to the Rabi frequency Ω_α and large detuning [12,16]. This strength can be positive or negative, corresponding to the zero phase or π phase of the standing wave. We expand the state vector as $|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\sum_{\alpha=1}^{2}|\psi_\alpha(t)\rangle|g_\alpha\rangle$ with $|\psi_\alpha(t)\rangle$ being the motional states entangling the corresponding internal pseudospin states $|g_\alpha\rangle$. Eq. (1) implies the correspondence between different $|\psi_\alpha(t)\rangle$ and different V_α. Therefore, the initial state can be a superposition of two external coherent states centred at different positions [12,39].

In the Lamb-Dicke approximation, $\cos(2kx) = 1 - 2\sin^2(2kx) \approx 1 - 2k^2x^2$, Eq. (1) is reduced to a kicked oscillator Hamiltonian with two components of the time-dependent "spring coefficient" $k_\alpha(t) = \omega^2 - \gamma_\alpha\sum_{j=1}^{\infty}\delta(t-jT)$, $\gamma_\alpha = \frac{4V_\alpha k^2}{m}$ for $\alpha = 1, 2$. The corresponding Schrödinger equation was solved by applying the trial-solution method [23,40,41] and a set of orthonormalized exact solutions was obtained as [38]

$$\begin{aligned}
\psi_{\alpha n}(x,t) &= \langle x|\psi_{\alpha n}(t)\rangle = R_{\alpha n}(x,t)e^{i\Theta_{\alpha n}(x,t)},\\
R_{\alpha n}(x,t) &= \left[\frac{\sqrt{c_{\alpha 0}}}{\sqrt{\pi}2^n n!\rho_\alpha(t)}\right]^{\frac{1}{2}} H_n(\xi_\alpha)e^{-\xi_\alpha^2/2},\\
\Theta_{\alpha n}(x,t) &= \frac{\dot\rho_\alpha(t)}{2\rho_\alpha(t)}x^2 + b_{\alpha 2}(t)x - \left(\frac{1}{2}+n\right)\theta_\alpha(t)\\
&\quad + \int\left[\frac{b_{\alpha 1}^2(t)}{2} - \frac{b_{\alpha 2}^2(t)}{2} + V_\alpha\sum_{j=1}^{\infty}\delta(t-jT)\right]dt,\\
\xi_\alpha(x,t) &= \frac{\sqrt{c_{\alpha 0}}}{\rho_\alpha(t)}x - \frac{b_{\alpha 1}(t)\rho_\alpha(t)}{\sqrt{c_{\alpha 0}}}, \quad n = 0,1,2\ldots.
\end{aligned} \tag{2}$$

Here, the trap frequency ω is rescaled by a reference frequency ω_0 which is in order of $10^5 \sim 10^8$ Hz [12], and time t and pulse period T are rescaled by ω_0^{-1}. Spatial coordinate x and probability density $|\psi_\alpha(x,t)|^2$ are in units of $x_0 = \sqrt{\hbar/(m\omega_0)}$ and its inverse x_0^{-1} respectively. The real functions $\rho_\alpha(t), \theta_\alpha(t), b_{\alpha 1}(t)$ and $b_{\alpha 2}(t)$ in Eq. (2) satisfy the relations

$$\begin{aligned}
\varphi_\alpha(t) &= \rho_\alpha(t)e^{i\theta_\alpha(t)} = A_{\alpha j}e^{i\omega t} + B_{\alpha j}e^{-i\omega t},\\
A_{\alpha j} &= A_{\alpha 0} - \frac{i\gamma_\alpha}{2\omega}\sum_{n=1}^{j}e^{-i\omega nT}\varphi_\alpha(nT),\\
B_{\alpha j} &= B_{\alpha 0} + \frac{i\gamma_\alpha}{2\omega}\sum_{n=1}^{j}e^{i\omega nT}\varphi_\alpha(nT),\\
b_{\alpha 1}(t) &= \frac{1}{\rho_\alpha^2}[b'_{\alpha 0}\,\mathrm{Re}(\varphi_\alpha) + b''_{\alpha 0}\,\mathrm{Im}(\varphi_\alpha)],\\
b_{\alpha 2}(t) &= \frac{1}{\rho_\alpha^2}[b''_{\alpha 0}\,\mathrm{Re}(\varphi_\alpha) - b'_{\alpha 0}\,\mathrm{Im}(\varphi_\alpha)]
\end{aligned} \tag{3}$$

in the time interval $0 \leqslant t < (j+1)T$, where $b'_{\alpha 0}$, $b''_{\alpha 0}$, $A_{\alpha j}$, $B_{\alpha j}$ are real constants, and the two integral constants $c_{\alpha 0} = \rho^2\dot\theta = \omega(A_{\alpha 0}^2 - B_{\alpha 0}^2)$ are obtained. We define the initial constant sets $S_\alpha = (b'_{\alpha 0}, b''_{\alpha 0}, A_{\alpha 0}, B_{\alpha 0})$ for $\alpha = 1, 2$ which determine the forms of the solutions, and one set S_α corresponds to a set of generalized coherent states $\psi_{\alpha n}(x,t)$. When $S_1 = S_2$ is set initially, the difference between solutions $\psi_{1n}(x,t)$ and $\psi_{2n}(x,t)$ can only be adjusted by the kick strengths γ_α. Inserting $\psi_{\alpha 0}(x,t)$ into the total state $|\Psi(x,t)\rangle$ can yield the superposition of two spatially separated harmonic oscillator coherent states with $\gamma_1 \neq \gamma_2$ and $S_1 \neq S_2$, which is the well-known Schrödinger cat state [31].

Now we apply a $\pi/2$ pulse of Ramsey type experiment to the ion, the state vector is rotated to the form [12]

$$|\Psi'_{nn'}(t)\rangle = \frac{1}{\sqrt{2}}\left[|\psi'_{1nn'}(t)\rangle|g_1\rangle + |\psi'_{2nn'}(t)\rangle|g_2\rangle\right], \tag{4}$$

with $|\psi'_{1nn'}\rangle = \frac{1}{\sqrt{2}}(|\psi_{1n}\rangle - |\psi_{2n'}\rangle)$ and $|\psi'_{2nn'}\rangle = \frac{1}{\sqrt{2}}(|\psi_{1n}\rangle + |\psi_{2n'}\rangle)$ for $n, n' = 0, 1, 2\ldots$ given by Eq. (2). In the rest of the article we only focus on the case of $n = n' = 0$, and we will leave out the subscript 0, namely $\psi_\alpha = \psi_{\alpha 0}$.

Here, we are interested in the connection between stability degree of the ground state $\Psi'(x,t)$ and the system parameters, which plays an important role in stabilizing the system by the adjustment of parameters. We define $\varphi_{\alpha j} = \varphi_\alpha(jT - 0^+)$ and $\varphi_{\alpha j+1} = \varphi_\alpha[(j+1)T - 0^+]$ and from Eq. (3) derive the linear mapping

$$\begin{aligned}
\begin{pmatrix}\varphi_{\alpha j+1}\\ \dot\varphi_{\alpha j+1}\end{pmatrix} &= M_\alpha\begin{pmatrix}\varphi_{\alpha j}\\ \dot\varphi_{\alpha j}\end{pmatrix},\\
M_\alpha &= \begin{pmatrix}\cos(\omega T) + \frac{\gamma_\alpha}{\omega}\sin(\omega T) & \frac{\sin(\omega T)}{\omega}\\ \gamma_\alpha\cos(\omega T) - \omega\sin(\omega T) & \cos(\omega T)\end{pmatrix}.
\end{aligned} \tag{5}$$