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# Effective moduli of multi-scale composites

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ABSTRACT

The effective moduli of a multi-scale composite are evaluated by a bottom-up (hierarchical) modeling approach. We focus on a two-scale structure in which the small scale includes a platelet array inside a matrix, and the large scale contains fibers inside a composite matrix. We demonstrate that the principal moduli of the multi-scale composite can be fine-tuned by the platelet arrangement and orientation. As a case study, we consider the phenomenon of fiber micro-buckling within the multi-scale composite. It is found that the compressive micro-buckling strength can be considerably increased for specific platelet orientations. The multi-scale design approach presented here can be used to generate novel families of composite materials with tunable mechanical properties.



## 1. Introduction

A multi-scale composite is a structure that includes reinforcements at different scales, bonded together by a matrix. Synthetic engineering composites are single-scale structures composed of fibers embedded in a matrix [18]. In recent years, attempts are made to improve these synthetic composites by including an additional reinforcement phase at a lower scale, for example by dispersing carbon nanotubes into their matrix. Such modifications significantly increase the composite strength and toughness but only slightly affect its moduli [21], which might be attributed to the randomness of the nanotubes orientation and dispersion. Natural materials such as hard tissues (e.g. bones, teeth) are examples of multi-scale composites with well defined structural arrangements throughout their different scales [23,16]. Such natural structures exhibit extraordinary mechanical properties, which normally originate from the structural properties at their smallest scale, often consisting of a staggered array of stiff platelets inside a soft matrix [14,7,5]. Synthetic configurations with well ordered small-scale structures could be obtained by methods such as layer-by-layer assembly [24], but are yet to be fully developed experimentally and mechanically understood at the analytical level. In some cases, the staggered structure may also affect the inelastic properties such as fracture toughness [20,2].

This work deals with a specific multi-scale composite configuration, the smallest scale of which consists of platelets (or rods) arranged in either regular (the platelets are perfectly parallel to each other) or staggered (the platelets are parallel but shifted), as fully defined later. The Halpin–Tsai model [9] can be used for evaluating the effective moduli of such structures. This model was obtained semi-empirically and the resultant expressions are functions of the platelet volume fraction and their cross sectional dimensions. The exact platelet arrangement however does not appear as a parameter in the Halpin–Tsai formulation. On the other hand, the Hopkins–Chamis model was designed for regular platelet arrays [10], and the expressions for the effective moduli are formulated by dividing the periodic unit into sub-regions and combining their stiffnesses using the Voigt and Reuss models [8]. The Hopkins–Chamis model however, cannot be applied to the staggered configuration due to the existence of non-negligible shear deformations [15,3]. Among the many mechanical models which have been proposed for the staggered micro-structure (e.g. [13,17,25], Gao's model [15] provides a compact formulation for the longitudinal modulus of the staggered array, designed for the asymptotic case of very soft matrix and small axial spacing between the platelets. Recently, Bar-On and Wagner [3,4] introduced a modified expression for the longitudinal modulus of the staggered array, which is applicable to a generic staggered geometry and coincides with Gao's formula for the corresponding asymptotic conditions.

The present study proposes an original bioinspired multi-scale composite structure made of a standard composite configuration (fiber–matrix) at the larger scale, and of an additional platelet (or rod) array at the lower scale. The effective moduli of such a multi-scale composite are evaluated by a bottom-up (hierarchical) modeling approach, and the effects of the small-scale reinforcement on the global mechanical properties are demonstrated for selected examples. As will be seen, the anisotropy of the lower scale array in the proposed multi-scale structure has a determining effect on the tuning capabilities of the global mechanical properties of the composite.

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## 2. Hierarchical modeling of multi-scale composites

A multi-scale composite is defined here as a structure in which each scale is a composite material, containing a matrix and a reinforcement. This work focuses on a two-scale composite, as shown in Fig. 1. The small-scale composite (SSC) includes a matrix reinforced by platelets (or rods), denoted by the subscripts m and p respectively. The large-scale composite (LSC) includes fibers embedded inside a matrix, denoted by the subscripts $f$ and $M$ respectively. The large-scale matrix is made of SSC layers possessing different angles. Denote $\phi_m$, $\phi_p$, $\phi_M$ and $\phi_f$, as the corresponding volume fractions, such that (in the absence of voids):

$$\phi_p + \phi_m + \phi_f = \phi_M + \phi_f = 1 \tag{1}$$

Assuming a uniform platelet distribution, the SSC platelet content is:

$$\tilde{\phi}_p = \phi_p/(\phi_p + \phi_m) \tag{2}$$

In the following we evaluate the effective moduli of the two-scale composite by a bottom-up (hierarchical) modeling approach. The moduli of the SSC are evaluated first by micro-mechanical and semi-empirical models. The large-scale matrix is then constructed from stacks of SSC layers, and the moduli of the LSC are finally extracted by using semi-empirical formulas.

### 2.1. The small-scale composite (SSC)

The SSC consists of a matrix and platelet (or rod) reinforcement, as shown in Fig. 2. The platelets are characterized by length $l$, height $h$ and thickness $t$ (whereas rods can be approximated as platelets with $h = t$). $e$ and $d$ are the longitudinal and transverse spacing between adjacent platelets, respectively. The directions of the SSC coordinates, $x_1$ and $x_2$, naturally correspond to the reinforcement longitudinal and transverse dimensions, $l$ and $h$ respectively. Two SSC configurations are considered: a regular array with a single platelet per periodic unit and a symmetrically staggered array with two equal overlaps per periodic unit (Fig. 2). Non-overlapping and non-symmetrical platelet arrangements are beyond the scope of this work. The matrix and platelets are assumed to be isotropic, with Young moduli $E_m$ and $E_p$ and shear moduli $G_m$ and $G_p$, respectively.

Focusing on planar elasticity, the SSC is characterized by four effective elastic moduli: longitudinal and transverse moduli ($E_{SSC11}$, $E_{SSC22}$), shear modulus ($G_{SSC12}$) and Poisson ratio ($\nu_{SSC12}$). These moduli are functions of the platelet material properties ($E_m$, $E_p$, $G_m$ and $G_p$), geometry ($l$, $h$ and $t$) and arrangement (regular or staggered). The moduli of the regular SSC are evaluated by the Hop-

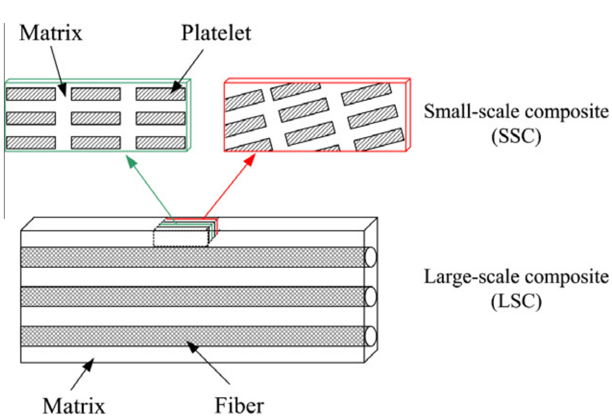


**Fig. 1.** A schematic view of a two-scale composite. The small-scale composite (SSC) includes platelets embedded inside a matrix. The large-scale composite (LSC) consists of long fibers inside a matrix made of SSC laminae with different angles.

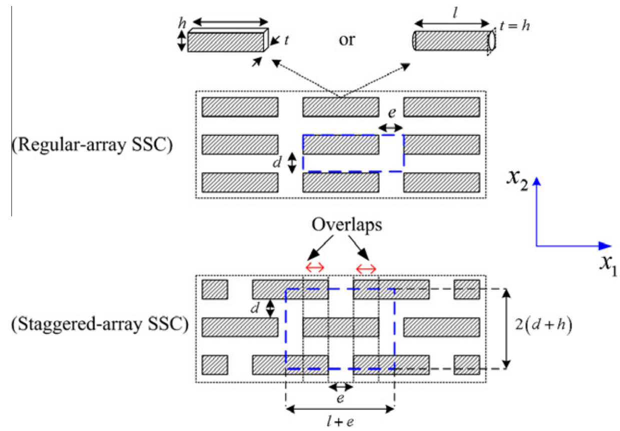


**Fig. 2.** A schematic description of regular and staggered SSCs, containing a matrix and platelets (or rods) with length $l$, height $h$ and thickness $t$. Rods are considered as platelets with $h = t$. $e$ and $d$ are the longitudinal and transverse spacing between adjacent platelets, respectively. The middle and bottom schemes show the regular and staggered SSC arrangements, where the area within the blue dashed lines represents the periodic unit for each configuration. $x_1$ and $x_2$ are the SSC coordinates.

kins–Chamis model [10], in which the unit cell is divided into sub-regions and its moduli are constructed by the Voigt and Reuss models. The longitudinal modulus is evaluated as [8]:

$$E_{SSC11} = E_m\left[1 - \Delta_h + \frac{\Delta_h}{1 - \Delta_l\left(1 - \frac{E_m}{\left[E_p\left[1 - \frac{\tanh(\gamma)}{\gamma}\right]\right]}\right)}\right] \quad \text{(regular)} \tag{3}$$

where $\Delta_h$ and $\Delta_l$ are the platelet thickness and length ratios:

$$\Delta_h = h/(h + d), \quad \Delta_l = l/(l + e) \tag{4a, b}$$

and $\gamma$ is the Cox shear-lag parameter of the regular array [3]:

$$\gamma = \frac{l}{h}\cdot\sqrt{2\frac{G_m}{E_p}\frac{h}{d}} = \rho_p\cdot\sqrt{2\frac{G_m}{E_p}\frac{\Delta_h}{1 - \Delta_h}} \quad \text{(regular)} \tag{5}$$

$\rho_p = l/h$ is the platelet aspect ratio. For the sake of simplicity, the shear-lag parameter here (5) was adapted from the standard Cox model [6], which usually underestimates its actual value for platelet geometry. More accurate shear-lag parameters can be found in the literature (e.g. [3,11]). Eqs. (3) and (5) can also be expressed by other non-dimensional parameters, the platelet content $\tilde{\phi}_p$ and the spacing ratio $\delta$:

$$\tilde{\phi}_p = \Delta_h\cdot\Delta_l, \quad \delta = \frac{d}{e} = \frac{1}{\rho_p}\frac{\tilde{\phi}_p(1 - \Delta_h)}{\Delta_h(\Delta_h - \tilde{\phi}_p)} \tag{6a, b}$$

The transverse and shear moduli, and Poisson ratio, are evaluated by:

$$E_{SSC22} = E_m\left[1 - \Delta_l + \frac{\Delta_l}{1 - \Delta_h(1 - E_m/E_p)}\right] \quad \text{(regular)} \tag{7}$$

$$G_{SSC12} = G_m\left[1 - \sqrt{\tilde{\phi}_p} + \frac{\sqrt{\tilde{\phi}_p}}{1 - \sqrt{\tilde{\phi}_p}(1 - G_m/G_p)}\right] \quad \text{(regular)} \tag{8}$$

$$\nu_{SSC12} = \nu_p\tilde{\phi}_p + \nu_m(1 - \tilde{\phi}_m) \quad \text{(regular)} \tag{9}$$

The transverse modulus is extracted similarly to the longitudinal one, by neglecting the shear-lag effects and replacing $\Delta_h \leftrightarrow \Delta_l$. The shear modulus uses $\tilde{\phi}_p$, which is invariant for an orthogonal rotation ($G_{SSC12} = {}_{SSC21}$). The Poisson ratio is simply evaluated by the rule-of-mixtures.