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Impact of two relaxation times on thermal, P and SV waves at interface with magnetic field and temperature dependent elastic moduli

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A B S T R A C T

In this article, two models of the generalized thermo-elastic theory are used to see the influence on the refraction and reflection of the plane waves at the interface under a constant magnetic field. The elasticity modulus depends on the reference temperature. The elasticity modulus is considered as a linear function of reference temperature. The resulting problem is solved by using the boundary conditions at the interface. The matrix equations have been solved numerically.

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Introduction

The infinite velocity of the thermal wave is used in the classical theory of thermo-elasticity. This assumption may be useful for many engineering problems, but practically it is unacceptable approximation. In some experiments finite speed of the thermal waves are observed, so to remove this difference generalized thermo-elastic theories LS and GL was proposed by Lord and Shulman [1]. Green and Lindsay [2] developed generalized thermo-elastic theory involving one thermal relaxation time. Lindsay and Green [2] derived a temperature dependent thermo-elasticity involving two relaxation times without violating the classical Fourier law of heat conduction. Because, propagation of wave in thermo-elastic media plays a vital role in several fields such as solid dynamics, earth quake engineering, nuclear reactors and aeronautic etc. Various authors considered the propagation of wave in thermo-elastic an isotropic medium. Parfitt and Eringen [3] considered the plane waves reflection from the flat wall of a micro-polar elastic half space. Ariman [4] studied the propagation of wave in a micro-polar elastic half space. For some relevant work of interest, we refer the readers to study the work of Kumar and Singh [5], Singh [6] and Deswal and Kumar [7]. But some papers described the influence of reference temperature elastic modulus. In this reference, Othman and Song [8] viewed the influence of temperature dependent elastic moduli on the reflection magneto thermo-elastic waves with two relaxation times.

Moreover, Abd-Alla et al. [9] considered the refraction and reflection of SV waves at the solid liquid interface by considering primary stress and three thermo-elastic theories. Kumar and Saini [10] illustrated the effect of refraction and reflection of waves at the interface between two different porous solids. Wei et al. [11] investigated the refraction and reflection of P waves at thermo-elastic and porous thermo-elastic medium.

The magneto thermo-elastic theory includes the impact of magnetic field on the thermo-elastic waves. This theory has achieved more importance in various industrial appliances, especially in nuclear devices. The connection of magnetic field with strain and thermal field has been discussed by many researchers; these include Sinha and Elsibai [12], Deresiewicz [13], Tuncay and Corapcioglu [14], Achenbach [15] and Z.D. Zhou et al. [16]. In this paper, we have considered with influence of two relaxation times on the refraction and reflection of thermo-elastic plane waves at the solid liquid interface. The refraction and reflection coefficient ratios of different refracted and reflected waves with the incident angle θ have been observed by Green Lindsay (GL) theory and dynamical coupling (CD) theory.

The current article is organized in the following order: Sectio n "Formulation of the problem" described the formulation of the problem. Method of solution is explained in Section "Methods of Solutions". Detail descriptions of the boundary conditions for the current scenario are given in Section "Boundary Conditions". Sect ion "Expressions for the refraction and reflection coefficients" is devoted to obtain the expressions for the refraction and reflection

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coefficients. Numerical results and discussion is given in Section " Numerical Results and Discussions".

Formulation of the problem

Let us assume an isotropic, linear, homogeneous, perfectly conducting and thermally elastic medium with temperature dependent mechanical characteristics covering at the interface of the two half-spaces.

We kept constant temperature T_0 throughout the body with uniform magnetic field $H_0 = (0, H, 0)$, which is applied in the positive direction of y-axis.

Basic equations

The electromagnetic field is controlled by the following Maxwell equations.

$$curl\ \circ = \mathbf{J} + \frac{\partial \mathcal{B}}{\partial \mathbf{t}}, \qquad \mathcal{B} = \epsilon_0 \mathbf{E}, \tag{1}$$

$$curl\ \mathbf{E} = -\mu_0 \frac{\partial \hbar}{\partial t}, \tag{2}$$

$$\boldsymbol{E} = -\mu_0 \left(\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H}_0 \right), \tag{3}$$

$$div\ \circ = 0. \tag{4}$$

Here E is an induced electric field, $\boldsymbol{H}_0$ is initial uniform magnetic intensity vector, ϵ_0 is electric permeability and J is the current density vector.

The generalized thermo-elastic differential equations under *GL* theory, in the absence of heat source and body force, has the form

1. Equation of motion

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + f_i \tag{5}$$

here f_i is the Lorentz force is given as under

$$f_i = \mu_0 (\mathbf{J} \times \mathbf{H}_0)_i \tag{6}$$

$$f_1 = -\mu_0 H \frac{\partial \hbar}{\partial x} - \epsilon_0 \mu_0^2 H^2 \ddot{u}, \quad f_2 = 0, \quad f_3 = -\mu_0 H \frac{\partial \hbar}{\partial z} - \epsilon_0 \mu_0^2 H^2 \ddot{w} \tag{7}$$

2. The constitutive law for the generalized thermo-elasticity theory under the GL theory has the form

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij} \left[\gimel \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) - \eta \left(T - T_0 + v_0 \frac{\partial T}{\partial t} \right) \right]. \tag{8}$$

3. Under GL theory, the heat conduction equation is

$$\mathrm{K} \nabla^2 T = \rho \tau_E \frac{\partial T}{\partial t} \left(1 + v_1 \frac{\partial}{\partial t} \right) + \eta T_0 \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right). \tag{9}$$

4. Strain-displacement relation

$$e_{ii} = u_{i,i}, \qquad e_{jj} = u_{j,j}, \qquad e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{10}$$

here $\gimel$, μ are lame's constants, K is thermal conductivity, ρ is density, τ_E is specific heat at constant strain, σ_{ij} is components of stress tensor, u_i is components of displacement vector, T is absolute temperature, t is time and v_0, v_1 are two relaxation times.

Where the derivative with respect to time is represented by a superposed dot and a comma after suffix shows material derivatives $i, j = x, z$.

The displacement components in two dimensional forms can be written as

$$u_x = u(x, z, t), \quad u_y = 0, \quad u_z = w(x, z, t). \tag{11}$$

Where, Helmolz's representations of the displacement components u_x and u_z in terms of scalar potential functions Φ and Ψ,

$$u = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial z}, \quad w = \frac{\partial \Phi}{\partial z} + \frac{\partial \Psi}{\partial x}. \tag{12}$$

We define temperature dependent parameters as follow:

$$\mathrm{E} = E_0 f(T), \ \gimel = \gimel_0 E_0 f(T), \quad \mu = \mu_0 E_0 f(T), \quad \eta = \eta_0 E_0 f(T) \tag{13}$$

The non-dimensional function of temperature is $f(T)$. When the modulus of elasticity is temperature independent then $f(T) = 1$ and $E = E_0$.

Putting, Eqs. (7), (8), (10) and (13) into Eq. (5) yield

$$\rho \frac{\partial^2 u_x}{\partial t^2} = E_0 f(T) \left[\gimel_0 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right) + 2\mu_0 \frac{\partial e_{xx}}{\partial x} - \eta_0 \frac{\partial}{\partial x} (T + v_0 \dot{T}) \right] + 2E_0 f(T) \mu_0 \frac{\partial e_{xz}}{\partial z} - \mu_0 H \frac{\partial \hbar}{\partial x} - \mu_0^2 H^2 \epsilon_0 \frac{\partial^2 u}{\partial t^2}, \tag{14}$$

$$\rho \frac{\partial^2 u_z}{\partial t^2} = E_0 f(T) \left[\gimel_0 \left(\frac{\partial^2 u}{\partial z \partial x} + \frac{\partial^2 w}{\partial z^2} \right) + 2\mu_0 \frac{\partial e_{zz}}{\partial z} - \eta_0 \frac{\partial}{\partial z} (T + v_0 \dot{T}) \right] + 2E_0 f(T) \mu_0 \frac{\partial e_{zx}}{\partial x} - \mu_0 H \frac{\partial \hbar}{\partial z} - \mu_0^2 H^2 \epsilon_0 \frac{\partial^2 w}{\partial t^2}. \tag{15}$$

Putting, Eq. (12) in Eqs. (1)–(4), we can obtain

$$\hbar = -H \nabla^2 \Phi. \tag{16}$$

We introduce the different non dimensional variables are follow:

$$x_i^* = \frac{x_i}{\omega_1 C_t}, \ u_i^* = \frac{u_i}{\omega_1 C_t}, \ t^* = \frac{t}{\omega_1 C_t}, \ v_0^* = \frac{v_0}{\omega_1}, \ v_1^* = \frac{v_1}{\omega_1}, \ \hbar^* = \frac{\hbar}{H},$$
$$\sigma_{ij}^* = \frac{\sigma_{ij}}{\rho C_t^2}, T^* = \frac{\eta_0 E_0 (T - T_0)}{\rho C_t^2}, \ \beta = 1 + \frac{c_a^2}{c^2}, \ \beta_1 = \frac{1}{1 - \beta^* T_0} = \frac{1}{f(T_0)}. \tag{17}$$

After non-dimensionalize, the Eqs. (8), (9), (14), (15) and (16) taken the following forms

$$\beta \beta_1 \frac{\partial^2 \Phi}{\partial t^2} = (1 + \beta_1 r_H) \nabla^2 \Phi - \left(\mathrm{T} + v_0 \frac{\partial T}{\partial t} \right), \tag{18}$$

$$\beta \beta_1 \frac{\partial^2 \Psi}{\partial t^2} = (1 - \alpha) \nabla^2 \Psi, \tag{19}$$

$$\nabla^2 \mathrm{T} = \left(\frac{\partial T}{\partial t} + v_1 \frac{\partial^2 T}{\partial t^2} \right) + \varepsilon \nabla^2 \dot{\Phi}, \tag{20}$$

$$\hbar = -\nabla^2 \Phi. \tag{21}$$

where ∇^2 is the Laplace's operator.

The constitutive equations reduce to

$$\beta_1 \sigma_{ij} = (1 - \alpha)(u_{i,j} + u_{j,i}) + \delta_{ij} \left((2\alpha - 1) \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) - \left(\mathrm{T} + v_0 \frac{\partial T}{\partial t} \right) \right). \tag{22}$$

Where, $\alpha = E_0(\gimel_0 + \mu_0)/\rho C_t^2$, $r_H = \frac{c_a^2}{C_t^2}$, $\varepsilon^* = \frac{\eta_0 T_0}{\rho^2 \tau_E C_t^2}$, $c_a^2 = \frac{\mu_0 H^2}{\rho}$, $C_t^2 = E_0(\gimel_0 + 2\mu_0)/\rho$, $c^2 = \frac{1}{\mu_0 \epsilon_0}$, $\omega_1 = K/\rho C_t \tau_E^2$.

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