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Viscous dissipation and Joule heating effects in MHD 3D flow with heat and mass fluxes

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ABSTRACT

The present research explores the three-dimensional stretched flow of viscous fluid in the presence of prescribed heat (PHF) and concentration (PCF) fluxes. Mathematical formulation is developed in the presence of chemical reaction, viscous dissipation and Joule heating effects. Fluid is electrically conducting in the presence of an applied magnetic field. Appropriate transformations yield the nonlinear ordinary differential systems. The resulting nonlinear system has been solved. Graphs are plotted to examine the impacts of physical parameters on the temperature and concentration distributions. Skin friction coefficients and local Nusselt and Sherwood numbers are computed and analyzed.

1. Introduction

The study of an electrically conducting fluid in view of its industrial and engineering applications is an important area of research for the investigators. It is well known fact that the final quality of product in manufacturing processes greatly depends on the cooling rate. A controlled cooling system is quite essential for such process. An electrically conducting fluid is the best candidate for applications in metallurgy, polymer technology and nuclear processes. In such processes the flow can be regulated by an applied magnetic field. Such applied magnetic field is quite useful to controlling momentum and heat transfer in boundary layer flows. Having such facts in mind, many authors have investigated the effect of applied magnetic field in flows through different flow configurations. Chen [1] investigated the effect of an applied magnetic field on the boundary layer flow of second grade and Walter's B liquids in the presence of thermal radiation. Analytical treatment of mixed convection flow of MHD viscoelastic fluid over a permeable stretching surface is made by Turkyilmazoglu [2]. Rashidi et al. [3] analyzed the entropy generation analysis in MHD slip flow of rotating viscous fluid with variable characteristics. An applied magnetic field effect on Falkner-Skan flow of Maxwell fluid via Chebyshev collocation method is examined by Abbasbandy et al. [4]. Seini and Makinde [5] discussed the magnetohydrodynamic boundary layer flow due to an exponentially stretching sheet. Hayat et al. [6] presented the series solutions of MHD three-dimensional flow of Maxwell fluid with variable thermal conductivity. Further recent investigations on MHD flows can be quoted through the studies [7–16].

The investigations through simultaneous effects of heat and mass transfer in the stretching flows are very popular subject amongst the recent researchers. Such analyses have potential role in the industrial and engineering processes like refrigeration and air conditioning, solar power collectors, damage of crops, desalination, human transpiration and many others [17–21]. Turkyilmazoglu [22] presented an analysis to study the combined effects of heat and mass transfer in the boundary layer flow of viscous nanofluid. Hayat et al. [23] developed the homotopic solutions for steady flow of Casson fluid with heat and mass transfer under thermal-diffusion and diffusion-thermo effects. Hayat and Alsaedi [24] studied the thermophoretic effects in boundary layer flow of an Oldroyd-B fluid with mixed convection. Shehzad et al. [25] extended the analysis of study [24] for a Jeffrey fluid model and provided the series solutions.

Joule heating is also known as ohmic heating. It is a mechanism by which the passage of an electric current through a conductor

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produces heat. Joule heating has a variety of usage in industrial and technological processes such as electric stoves, electric heaters, incandescent light bulb, electric fuses, electronic cigarette, thermistor, food processing and several others. Thus Rahman [26] examined convective flow of micropolar fluid past a vertical radiate isothermal permeable surface with viscous dissipation and Joule heating. Hayat et al. [27] discussed non-uniform melting heat transfer effects in boundary-layer flow of nanofluid with viscous dissipation and Joule heating. Sheikholeslami and Ganji [28] studied magnetohydrodynamic flow of nanofluid between two parallel vertical permeable sheets with viscous dissipation and Joule heating. Mahanthesh et al. [29] addressed unsteady magnetohydrodynamic three-dimensional flow of Eyring-Powell nanofluid with viscous dissipation and Joule heating. Khan et al. [30] examined chemically reactive flow of micropolar fluid with viscous dissipation and Joule heating.

The cheaper raw materials are converted into higher standard products in chemically industrial processes. Such chemical reactions are made in a reactor. To provide a suitable environment to finishing products, a chemical reaction plays major role. Chemical reaction effects are quite prominent in petroleum reservoirs, chemical catalytic reactors, nuclear waste repositories, spreading of chemical pollutants, diffusion of medicine in blood veins etc. Hayat et al. [31] investigated the unsteady flow of third grade fluid over a stretching surface in the presence of chemical reaction. MHD mixed convection flow past a vertical porous plate embedded in a porous medium with thermal radiation and chemical reaction is examined by Makinde [32]. Rashidi et al. [33] discussed the group theoretic and differential transform analyses of mixed convective heat and mass transfer from a horizontal surface with chemical reaction. Effects of mass transfer on MHD flow of Casson fluid with chemical reaction and suction is examined by Shehzad et al. [34]. Mukhopadhyay and Vajravelu [35] studied the diffusion of chemically reactive species in Casson fluid flow by an unsteady permeable stretching surface.

The prime objective of present attempt is to explore the magnetohydrodynamic (MHD) three-dimensional flow of viscous fluid in the presence of chemical reaction, Joule heating and viscous dissipation. Flow is induced due to an exponentially stretching surface. Prescribed heat flux (PHF) and prescribed concentration flux (PCF) conditions are taken at the surface. To the best of our knowledge, no such consideration for three-dimensional flow of viscous fluid is given in the literature yet. Mathematical formulation is presented for both prescribed heat flux (PHF) and prescribed concentration flux (PCF) conditions (see [36–40]). Homotopy analysis method (HAM) [41–50] is applied for the solution development. Infact homotopy analysis method (HAM) has three advantages. Firstly it does not require small/large physical parameters in the problem. Secondly it provides a simple way to ensure the convergence of series solutions. Thirdly it provides a large freedom to choose the base functions and related auxiliary linear operators. Plots of physical quantities are presented and discussed. Further the skin-friction coefficients and local Nusselt and Sherwood numbers are computed and analyzed.

2. Mathematical modeling

We consider the steady three-dimensional flow of an incompressible viscous fluid. The fluid is caused by an exponentially stretching surface. Magnetic field of strength B_0 is applied in the z-direction. Induced magnetic field is not considered for small magnetic Reynolds number. A Cartesian coordinate system is chosen in such a manner that x- and y-axes are taken along the stretching surface and z-axis is normal to it. The sheet at $z = 0$ is stretched in the x- and y-directions with velocities U_w and V_w respectively. Chemical reaction, viscous dissipation and Joule heating effects are taken into account. The thermophysical properties of fluid are taken constant. The governing boundary-layer expressions for three-dimensional flow of viscous fluid in the absence of thermal radiation are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu\frac{\partial^2 u}{\partial z^2}-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\nu\frac{\partial^2 v}{\partial z^2}-\frac{\sigma B_0^2}{\rho}v, \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2}+\frac{\sigma B_0^2}{\rho c_p}(u^2+v^2)+\frac{\mu}{\rho c_p}\left(\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right), \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D\frac{\partial^2 C}{\partial z^2}-K_1(C-C_\infty). \tag{5}$$

The boundary conditions for the present flow analysis are

$$u=U_w=U_0e^{\frac{x+y}{L}},\ \ v=V_w=V_0e^{\frac{x+y}{L}},\ \ w=0,\quad \text{at } z=0;$$
$$u\to 0,\ \ v\to 0\quad \text{when } z\to\infty. \tag{6}$$

The boundary conditions for the prescribed heat flux (PHF) and prescribed concentration flux (PCF) are imposed as follows:

$$\text{PHF}: -k\left(\frac{\partial T}{\partial z}\right)_w=T_0e^{\frac{(A+1)(x+y)}{2L}}\quad \text{at } z=0 \text{ and } T\to T_\infty\quad \text{when } z\to\infty, \tag{7}$$

$$\text{PCF}: -D\left(\frac{\partial C}{\partial z}\right)_w=C_0e^{\frac{(B+1)(x+y)}{2L}}\quad \text{at } z=0 \text{ and } C\to C_\infty\quad \text{when } z\to\infty, \tag{8}$$

where u, v and w are the velocity components in the x-, y- and z-directions respectively, $\nu=\mu/\rho$ the kinematic viscosity, μ the dynamic viscosity, ρ the density of fluid, σ the electrical conductivity, T the temperature, $\alpha=k/\rho c_p$ the thermal diffusivity of the fluid, k the thermal conductivity, c_p the specific heat at constant pressure, C the concentration, D the diffusion coefficient, K_1 the reaction rate, U_0, V_0, T_0 and C_0 the constants, L the reference length, A the temperature exponent, B the concentration exponent, T_w and T_∞ the temperatures of the surface and far away from the surface and C_w and C_∞ the concentrations at the surface and far away from the surface. The subscript w denotes the wall condition. The dimensionless variables can be defined as

$$\left.\begin{aligned} &u=U_0e^{\frac{x+y}{L}}f'(\eta),\ v=U_0e^{\frac{x+y}{L}}g'(\eta),\ w=-\left(\tfrac{\nu U_0}{2L}\right)^{1/2}e^{\frac{x+y}{2L}}(f+\eta f'+g+\eta g'),\\ &T=T_\infty+\tfrac{T_0}{k}\sqrt{\tfrac{2\nu L}{U_0}}e^{\frac{A(x+y)}{2L}}\theta(\eta),\ C=C_\infty+\tfrac{C_0}{D}\sqrt{\tfrac{2\nu L}{U_0}}e^{\frac{B(x+y)}{2L}}\phi(\eta),\ \eta=\left(\tfrac{U_0}{2\nu L}\right)^{1/2}e^{\frac{x+y}{2L}}z.\end{aligned}\right\} \tag{9}$$

Eq. (1) is automatically satisfied and Eqs. (2)–(8) have the following forms

$$f'''+(f+g)f''-2(f'+g')f'-M^2f'=0, \tag{10}$$

$$g'''+(f+g)g''-2(f'+g')g'-M^2g'=0, \tag{11}$$

$$\theta''+\Pr\left((f+g)\theta'-A(f'+g')\theta+M^2Ec\left(f'^2+g'^2\right)+Ec\left(f''^2+g''^2\right)\right)=0, \tag{12}$$