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Application of discontinuous Galerkin method for solving a compressible five-equation two-phase flow model

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ABSTRACT

In this article, a reduced five-equation two-phase flow model is numerically investigated. The formulation of the model is based on the conservation and energy exchange laws. The model is non-conservative and the governing equations contain two equations for the mass conservation, one for the over all momentum and one for the total energy. The fifth equation is the energy equation for one of the two phases that includes a source term on the right hand side for incorporating energy exchange between the two fluids in the form of mechanical and thermodynamical works. A Runge-Kutta discontinuous Galerkin finite element method is applied to solve the model equations. The main attractive features of the proposed method include its formal higher order accuracy, its nonlinear stability, its ability to handle complicated geometries, and its ability to capture sharp discontinuities or strong gradients in the solutions without producing spurious oscillations. The proposed method is robust and well suited for large-scale time-dependent computational problems. Several case studies of two-phase flows are presented. For validation and comparison of the results, the same model equations are also solved by using a staggered central scheme. It was found that discontinuous Galerkin scheme produces better results as compared to the staggered central scheme.

Introduction

In two-phase flows, two fluids of different densities are separated by a thin interface (see Fig. 1). The flow can be incompressible or compressible. Phases are identified as "homogeneous" parts of the fluid for which unique local state and transport properties can be defined. Generally, phases are considered as the state of matter, e.g. gas/vapor, liquid, or solid. The flow of gas carrying liquid droplets or solid particles or the flow of liquid carrying vapor or flow of solid granular material and fluid or gas bubbles are the typical examples of two-phase flows. Normally, in the case of two-phase flows, we are not interested in a detailed description of particle interaction, instead we want to describe the flow as a whole. This is exactly the situation where the homogenized approach comes into play. An important issue concerning the systems of governing equations for two-phase flow models is that they are intrinsically non-conservative. The mathematical structures of the non-conservative systems are more complicated as compared to conservation laws. Also, there is a lack of theory for numerical methods to solve such systems. On the other hand, the development of efficient numerical methods for the solution of two-phase flows is of great importance. As the model equations are intrinsically non-conservative, one has to provide non-conservative methods for their solutions.

Two phase flows can be observed in nature very easily, such as rainy or snowy winds, avalanches, debris flows, tornadoes, typhoons, air and water pollution, volcanic activities, and so on. They are also working processes in a variety of conventional and nuclear power plants, combustion engines, propulsion systems, oil and gas transport, chemical industry, biological industry, process technology in the metallurgical industry or in food production, blood flow, and etc. Due to their wide range applications, two-phase flows require suitable mathematical models to predict their physical behavior. However, modeling and simulation of such flows are challenging tasks.

Methods of averaging have been in use since the mid-70s when Ishii [1] presented the governing equations for the homogenized flow in his classical book. Nowadays, the more or less established basic model includes the two continuity, two momentum, and two energy equations for both phases. The averaging of the single phase equations results in additional terms, which describe the

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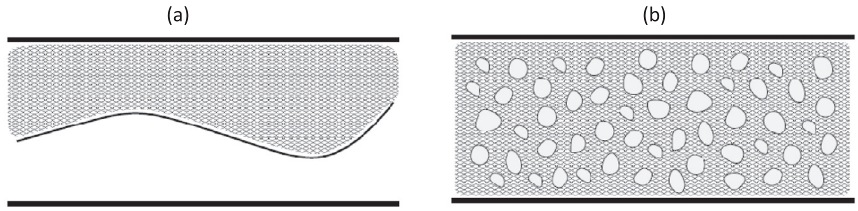

Fig. 1. Schematic diagram for two types of flows. (a) Flows separated by sharp interface. (b) Multiphase flows.

interaction between the phases. These are the mass transfer terms for the continuity equations, the momentum exchange terms for the momentum equations, and the energy exchange terms for the energy equations. The closure to the system of governing equations is usually achieved by adding an additional equation for the fraction of one of the phases, and the equations of state for both phases. Another approach is to use some simplifying assumptions, like incompressibility of one phase, equality of pressures, and etc.

Several two-phase flow models exist in the literature for describing the behavior of physical mixtures. For each fluid, they contain separate pressures, velocities and densities. If a convection equation for the interface motion is coupled with the conservation laws, the models are called as seven-equation models. One of such models for solid-gas two-phase flows was initially introduced by Baer and Nunziato [2] and it was further investigated by Abgrall and Saurel [3,4]. These seven equation models are considered as the best and established two-phase flow models. However, they have a number of numerical complexities. To resolve these difficulties researchers have proposed reduced three to six-equation models [5–7].

Kapila deduced a five equation model [5] from Baer and Nunziato's seven-equation model [2] and it is a well known reduced model that has been successfully implemented to study interfacing compressible fluids, barotropic and non-barotropic cavitating flows. The Kapila's five-equation model contains first four conservative equations, two for the mass conservation of both fluids, one for the total momentum conservation of the mixture and one for the total energy conservation. The fifth equation is a non-zero convection equation for the volume fraction of one of the two phases.

Although, Kapila's five equation model is simple, but it has a number of serious difficulties. For example, the model is still non-conservative and, thus, it is difficult to obtain a numerical solution which converges to the physical solution. Another issue is related to non-conservative behavior of the mixture sound speed [8].

In order to make the Kapila's five-equation model easier and to remove the aforementioned difficulties, Kreeft and Koren [6] have introduced a new formulation of the Kapila's five equation model. This new model is also non-conservative and it contains five equations [6]. The first two equations are for the conservation of mass, one for the mixture momentum conservation and one for the total energy conservation. The fifth equation is the energy equation for one of the two phases which includes a source term on the right hand side representing the energy exchange between two fluids in the form of mechanical and thermo-dynamical work. The two-phase flow models have already been solved by finite volume type schemes, such as central upwind scheme, central NT scheme, space-time CESE scheme and kinetic flux vector splitting (KFVS) scheme [9–13]. Also, diffuse interface method and finite volume WENO scheme have been used to solve the two-phase flow models [14–16].

The discontinuous Galerkin (DG) finite element method was initially introduced by [17] for solving neutron transport equations. Afterwards, various DG methods were developed and formulated by Cockburn and Shu for nonlinear hyperbolic system in the series of papers, see for example [18–20]. DG-methods are being applied in the main stream of computational fluid dynamic models, see for example [21–25]. The DG methods are versatile, flexible, and have intrinsic stability making them suitable for convection dominated problems. DG-methods can be efficiently applied to partial differential equations (PDEs) of all kind including equations whose type changes within the computational domain.

DG-methods belong to the class of finite element method (FEM) which have several advantages over finite difference methods (FDMs) and finite volume methods (FVMs). For instance, they inherit geometric flexibility of FVMs and FEMs, retain the conservation properties of FVMs, and possess high-order properties of FEMs. Therefore, DG-methods are locally conservative, stable, and high order accurate. These methods satisfy the total variation bounded (TVB) property that guarantees the positivity of the schemes, see e.g. [18–20]. In contrast to high order FDMs and FVMs, DG-methods require a simple treatment of the boundary conditions in order to achieve high order accuracy uniformly. Moreover, DG methods allow discontinuous approximations and produce block-diagonal mass matrices that can be easily inverted through algorithms of low computational cost. These methods incorporate the idea of numerical fluxes and slope limiters in a very natural way to avoid spurious oscillations (wiggles), which usually occur due to shocks, discontinuities or sharp changes in the solution.

In this paper, Runge-Kutta DG-scheme of order two is implemented for solving the reduced five-equation model of Kreeft and Koren [6,18–20]. The scheme employs a DG-method in the space-coordinate that converts the given system of partial differential equations to a system of ordinary differential equations (ODEs). The resulting ODE-system is then solved by using explicit and nonlinearly stable high order Runge-Kutta method. To guarantee the positivity of the numerical scheme an additional TVB property of the proposed ODE-solver along with the RK-DG is used. The numerical test problems of this manuscript verify the accuracy and efficiency of the current DG-scheme for solving two-phase flow models. For validation, the numerical results of the proposed scheme are compared with those obtained from the staggered central NT scheme [26].

The present article is organized as follows. Section "Compressible two-phase flow model" is devoted to the introduction of one-dimensional compressible two-phase flow model of Kreeft and Koren [6]. The discontinuous Galerkin method is presented in Section "Discontinuous Galerkin method for compressible TPSF model". Numerical case studies are carried out in Section "Numerical test problems". Finally, concluding remarks are given in Section "Conclusions".

Compressible two-phase flow model

In this section, the one-dimensional reduced two-phase flow model of Kreeft and Koren [6] is presented. The considered model