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Some new exact solitary wave solutions of the van der Waals model arising in nature

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ABSTRACT

This work proposes two well-known methods, namely, Exponential rational function method (ERFM) and Generalized Kudryashov method (GKM) to seek new exact solutions of the van der Waals normal form for the fluidized granular matter, linked with natural phenomena and industrial applications. New soliton solutions such as kink, periodic and solitary wave solutions are established coupled with 2D and 3D graphical patterns for clarity of physical features. Our comparison reveals that the said methods excel several existing methods. The worked-out solutions show that the suggested methods are simple and reliable as compared to many other approaches which tackle nonlinear equations stemming from applied sciences.

Introduction

Nonlinear Partial differential equations NPDEs describe a number of physical and complex phenomena comprising solid mechanics, quantum field theory, acoustic waves in crystals, biophysics, optical fibers, biology, nonlinear optics and plasma physics etc. A number of powerful methods have been formulated and implemented to solve NPDEs. Generally, a wave transformation is used to convert the equation under consideration into a nonlinear ODE which leads for further solution. Numerous effective approaches, such as Hirota's bilinear method [1], Variational iteration method [2], Exp $(-\varphi(\eta))$-expansion method [3,4], $\left(\frac{G'}{G}\right)$-expansion method [5], Exp-function method [6–8], Improved tan $\left(\frac{\phi}{2}\right)$-expansion method [9–11], Modified simple equation method [12], Exponential rational function method [13], Trial equation method [14], Jacobi elliptic function expansion method [15], Tanh-Coth method [16], Extended trial equation method [17–24], Sine-cosine method [25,26], Mapping method [27], Auxiliary equation method [28,29], Kudryashov method [30], F-expansion method [31,32], Ansatz method [33], Riccati equation mapping method [34], Khater method [35] and others are applied to obtain solutions of NPDEs.

Let us consider the van der Waals normal form as follows [36]:

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u}{\partial x^2} - \eta\frac{\partial u}{\partial t} - u^3 - \varepsilon u\right) = 0. \tag{1}$$

where u describes the field which reflect correction to critical average vertical density, x is the horizontal direction of the granular system. ε and h are the bifurcation parameter and effective viscosity respectively. The latter is proportional to the compressibility coefficient. This equation of fluidized granular matter is one of the most significant nonlinear PDEs, possessing key role in industrial applications, pharmaceuticals, civil engineering and geophysics. Further, the model describes the phase separation phenomenon. Granulation is the process which converts primary powder particles into larger form of particles whose result is called granules. A mixture of separated solid, macroscopic particles is said to be granular matter and the particles lose energy when they interact. Friction is one of the common examples of granulation as a consequence of particle collision. Granular solids and granular gasses are two main kinds of granular matter. The former describes the matter in which the grains are quite stationary as compared to each other when the average energy of each grains is less and the latter gives description of the matter in which the contact among the grains is very rare. Sand, rice, coat, nuts, snow, coffee, fertilizer, corn flakes, and ball bearings are some example of granular material. Also, powders are special cases of granular matter for being a mixture of micro size particles. According to Patrick Richard (a material scientist),

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granular materials appear in many physical phenomena and have a wide range of applications in material industry. In order to discuss the physical characteristics of this model [36–38], modified simple equation method [39,40], modified extended tanh-function method [41,42], novel $(\frac{G'}{G})$-expansion method [43,44], $\exp(-\varphi(\xi))$-expansion method [4,45–47], the improved $(\frac{G'}{G})$-expansion method [48] etc have been employed.

In this paper, numerous new exact solutions for the van der Waals normal form for the fluidized granular matter are attained by applying exponential rational function method (ERFM) and Generalized Kudryashov method (GKM). The proposed methods are very advantageous for providing new exact traveling wave solutions and are very beneficial, easy to use and straightforward.

This article contains following sections: In Section "Explanation of the methods", we briefly describe the aforementioned methods. In Section "Application", the applications of the said methods are discussed for solution of the van der Waals normal form for the fluidized granular matter. Physical description is presented in Section "Physical interpretation" which is finally followed by the Conclusion in Section 5.

Explanation of the methods

Exponential rational function method [13,49]

Consider the following nonlinear PDE

$$S(u, u_x, u_x, u_y, u_z, u_{xx}, \ldots..) = 0, \tag{2}$$

S being a polynomial of u and $u, u_x, u_x, u_y, u_z, u_{xx}, \ldots$ are its partial differential operators of the unknown function u. This method contains main steps as follows [13,49]:

Step 1: Consider following transformation

$$u(x,t) = U(\xi), \quad \xi = kx + lt. \tag{3}$$

where k, and l denote constants having arbitrary values, so that Eq. (1) can be transformed into the ODE

$$T(U, U', U'', U''', \ldots) = 0. \tag{4}$$

Here $'$ denotes derivative in terms of ξ.

Step 2: The aforesaid technique assumes the exact solution of Eq. (4) as

$$U(\xi) = \sum_{n=0}^{N} \frac{a_n}{(1+e^{\xi})^n}. \tag{5}$$

where a_n are constants whose values are to be known. The balancing integer M can be known by implementing homogenous balance principle.

Step 3: Plugging Eq. (5) in Eq. (4) and gathering up all the terms having equal order of $e^{i\xi}, i = 1, 2, \ldots, 4$, we form another polynomial in $e^{i\xi}$ from left hand side of Eq. (4). A set of algebraic equations is formed for a_n by equalizing coefficients of polynomial to zero. On working out the system, we come up with various exact solutions of (1).

Generalized Kudryashov method [49,50]

Consider the following nonlinear PDE

$$S(u, u_x, u_x, u_y, u_z, u_{xx}, \ldots..) = 0, \tag{6}$$

S being a polynomial of unknown function u and $u, u_x, u_x, u_y, u_z, u_{xx}, \ldots$ are its partial differential operators. This method contains main steps as follows [50,51]

Step 1: Consider following transformation

$$u(x, y, t) = U(\xi), \quad \xi = x - \omega t,$$

for converting Eq. (1) into the ODE

$$T(u, u', u'', u''', \ldots) = 0. \tag{7}$$

Here $'$ denotes derivative in terms of ξ and $\omega \neq 0$ being real number, representing relative wave mode velocity. Eq. (7) can be integrated more than one time unless the resulted expression becomes simplest, while letting constant of integration to be zero.

Step 2: The aforesaid technique assumes the exact solution of Eq. (7) as

$$u(\xi) = \frac{\sum_{i=0}^{N} a_i Q^i(\xi)}{\sum_{j=0}^{M} b_j Q^j(\xi)}, \tag{8}$$

where a_i and b_j are constants to be calculated and $i = 0, 1, 2, \ldots\ldots, N$, $j = 0, 1, 2, \ldots\ldots, M$. Further, $a_N \neq 0$, $b_M \neq 0$ and we also note that $Q = Q(\xi)$ satisfying the ODE

$$\frac{dQ(\xi)}{d\xi} = Q^2(\xi) - Q(\xi), \tag{9}$$

Eq. (9) has solution of the form as follows

$$Q(\xi) = \frac{1}{1 + A\, exp(\xi)}, \tag{10}$$

In the above expression, A being an integration constant.

Step 3: By using homogeneous balance principle, we find integers N and M which appear in Eq. (8). Degree of $u(\xi)$ as $D(u(\xi)) = N - M$ are defined which yields the degree of other expression

$$\begin{aligned} D\left(\frac{d^q u}{d\xi^q}\right) &= N - M + q, D\left(u^p \left(\frac{d^q u}{d\xi^q}\right)^s\right) \\ &= (N-M)p + s(N - M + q), \end{aligned} \tag{11}$$

p, s, and q being integers.

Step 4: Using Eqs. (8) and (9) into Eq. (7), we get a polynomial in Q^{i-j}, leading toward a system of algebraic equations by equalizing coefficients of like powers of Q to zero. We solve this system by using Maple for finding the parameters a_i, b_j and ω and finally, we construct exact solutions of Eq. (1).

Application

In this section we use exponential rational function method and Generalized Kudryashov method on the van der Waals normal form for the fluidized granular matter.

Implementation of exponential rational function method

Consider the following transformation

$$u(x,t) = U(\xi), \quad \xi = kx - \omega t, \tag{12}$$

where k being the wave number, and ω is angular frequency.

Using the transformation (12) in Eq. (1) will give us following nonlinear ODE

$$\frac{(\omega^2 - \varepsilon k^2)}{k^2} U'' - 6UU'^2 - 3U^2U'' + \eta\omega U''' + k^2 U^4 = 0, \tag{13}$$

Integration of Eq. (13) twice will give us

$$\begin{aligned} &\frac{(\omega^2 - \varepsilon k^2)}{k^2} U - U^3 + \eta\omega U' + k^2 U'' \\ &\quad = 0, \quad \text{(letting constant of integration to be zero)} \end{aligned} \tag{14}$$