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# Non-Darcy flow of water-based carbon nanotubes with nonlinear radiation and heat generation/absorption

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Here modeling and computations are presented to introduce the novel concept of Darcy-Forchheimer three-dimensional flow of water-based carbon nanotubes with nonlinear thermal radiation and heat generation/absorption. Bidirectional stretching surface induces the flow. Darcy's law is commonly replace by Forchheimer relation. Xue model is implemented for nonliquid transport mechanism. Nonlinear formulation based upon conservation laws of mass, momentum and energy is first modeled and then solved by optimal homotopy analysis technique. Optimal estimations of auxiliary variables are obtained. Importance of influential variables on the velocity and thermal fields is interpreted graphically. Moreover velocity and temperature gradients are discussed and analyzed. Physical interpretation of influential variables is examined.



## Introduction

A carbon nanotube (CNTs) is a tube shaped material, allotropes of carbon with a cylindrical nanostructure. These carbon molecules in cylindrical shape have exceptional characteristics, which are beneficial for nanotechnology, optics, electronics and other fields of materials science and engineering. Owing to the material's excellent strength and stiffness the cylindrical nanotubes are established with length-to-diameter ratio up to 132,000,000 remarkably higher when compared with other material. Carbon nanotubes have extensive applications in different fields for example in tissue engineering, prostheses, genomics, pharmacogenomics, drug delivery, surgery and general medicine etc. Carbon nanotubes can be categorized into two subclasses. These depend on structure of material, namely single wall carbon nanotubes (SMCNTs) and multi wall carbon nanotubes (MWCNTs). Thermal conductivity enhancement through nanotube suspension is examined by Choi et al. [1]. They considered oil based nanoliquids comprising carbon nanotubes and found that nanotubes yield remarkable thermal conductivity enhancement. A model based on Maxwell theory valid for carbon nanotubes (CNTs) characteristics transport is presented by Xue [2]. Non-Fourier heat flux and unsteady chemically reactive flow through SWCNTs and MWCNTs is investigated by Hayat et al. [3]. They considered Xue model for the effective thermal conductivity of nanoliquid. They found that Nusselt number is enhanced for large thermal relaxation and curvature parameters. MHD flow of carbon water nanomaterial by a stretchable disk with Marangoni convection and Rosseland approximation is explored by Mahanthesh et al. [4]. They used Runge-Kutta method via shooting technique to find out the computational results of nonlinear expressions. Their results illustrated that heat transfer rate increases for higher Marangoni number and nanoparticles volume fraction. However it declines for magnetic variable. MHD slip flow with convective heat transport in presence of SWCNTs and MWCNTs is analyzed by Haq et al. [5]. Recently few meaningful attempts for flows with SWCNTs and MWCNTs have been presented these studies [6–10].

Flow through porous space have extensive applications in various fields like petroleum engineering, industries and geothermal operations. Flow regime in porous space is commonly characterized by a dimensionless number (Reynolds number). Darcy's law is valid to describe flow in porous space at low flow rates i.e., $(Re < 1)$ (when flow rate and pressure gradient have linear relationship). This law predicts that viscous forces dominate over inertial forces in porous space. Mostly flow in porous space is described by Darcy's law, this law is not adequate for high flow rates. For higher flow rates the Forchheimer relation is used. Infact

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**Nomenclature**

| | | | |
|---|---|---|---|
| $u, v, w$ | velocity components | $T_w$ | surface temperature |
| $C_p$ | specific heat | $T_\infty$ | ambient temperature |
| $F_I$ | Forchheimer parameter | $\kappa_{fh}$ | Forchheimer permeability |
| $\nu_{nf}$ | kinematic viscosity | $\mu_{nf}$ | dynamic viscosity |
| $\sigma^*$ | Stefan Boltzman constant | $\rho_{nf}$ | density of nanofluid |
| $(\rho c_p)_{nf}$ | heat capacity | $k_{nf}$ | Effective thermal conductivity |
| $q_w$ | heat flux | $Nr$ | radiation parameter |
| $\lambda$ | heat generation/absorption parameter | $k^*$ | mean absorption coefficient |
| $\mu_f$ | dynamic viscosity | $\rho_f$ | density of fluid |
| Pr | Prandtl number | $\mathrm{Re}_x$ | Reynold number |
| $\omega$ | porosity parameter | $\theta_w$ | temperature ratio variable |
| $\phi$ | volume fraction of nanomaterial | $Nu_x$ | Nusselt number |
| $C_{fx}$ | skin friction coefficient | $q_r$ | radiative flux |
| $\tau_w$ | shear stress | $k$ | thermal conductivity parameter |
| $nf$ | nanofluid | | |

Forchheimer [11] introduced a new nonlinear contribution of velocity which is called Forchheimer term. Hayat et al. [12] investigated Darcy Forchheimer flow of ferromagnetic second grade fluid over a stretchable sheet. Some recent investigations about Darcy-Forchheimer flow can be seen in the studies [13–20].

In view of fast development of human society, numerous energy problems emerge for example environmental pollution and storage of global energy. Engineers and scientists are engaged in modeling new resources for sustainable energy. Solar energy is best source of renewable energy which offers a solution to this issue. Heat transport subject to Rosseland approximation has many applications in engineering, physics, nuclear plants and space technology, aerodynamic rockets, solar power technology, gas cooled nuclear reactors, counting combustion, furnace design, nuclear reactor protection and photo chemical reactors etc. Cortell [21] initially examined radiative flow over a stretchable surface. Sheikholeslami et al. [22] explored MHD flow of nanomaterial in subject to thermal radiation. Reddy et al. [23] studied impact of nonlinear radiation MHD flow of ferroliquids with temperature dependent viscosity. Few modern investigations on this topic can be mentioned in Refs. [24–30].

The purpose of present attempt is to model three-dimensional flow subject to SWCNTs and MWCNTs, Darcy-Forchheimer relation for porous space is considerd. Heat transfer process is explored subject to nonlinear thermal radiation and heat generation/absorption. Outcomes of SWCNT and MWCNT with water as base fluid are achieved and compared. Xue model [2] of nanomaterial is employed. The resulting nonlinear expressions are solved by Optimal homotopy analysis method (OHAM) [31]. Heat transfer rate and surface drag forces are computed and discussed.

## Problem statement

Three-dimensional Darcy-Forchhiemer flow of water and carbon nanotubes are considered. Both single (SWCNTs) and multiple (MWCNTs) walls carbon nanotubes are studied. Flow is due to bidirectional stretchable surface. Heat transport process is examined through nonlinear thermal radiation and heat absorption/generation. Xue [2] model for nanoliquid transport is implemented. In cartesian coordinates system, the flow equations are governed by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu_{nf}\frac{\partial^2 u}{\partial z^2}-\frac{\nu_{nf}}{\kappa_{fh}}u-F_0u^2, \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\nu_{nf}\frac{\partial^2 v}{\partial z^2}-\frac{\nu_{nf}}{\kappa_{fh}}v-F_0v^2, \tag{3}$$

$$(\rho c_p)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=k_{nf}\frac{\partial^2 T}{\partial z^2}-\frac{\partial q_r}{\partial z}+Q_0(T-T_\infty), \tag{4}$$

with

$$u=U_w(x)=ax,\quad v=V_w(y)=by,\quad w=0,\quad T=T_w,\quad \text{at } z=0$$
$$u=0,\quad v=0,\quad T\to T_\infty,\quad \text{at } z\to\infty. \tag{5}$$

The theoretical model for nanoliquid transport proposed by Xue [2] gives:

$$\left.\begin{array}{l}\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},\ \nu_{nf}=\frac{\mu_{nf}}{\rho_{nf}},\ \rho_{nf}=\rho_f(1-\phi)+\rho_s\phi,\\ (\rho c_p)_{nf}=(\rho c_p)_f(1-\phi)+(\rho c_p)_s\phi,\ \frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+2\phi(k_f-k_s)}.\end{array}\right\} \tag{6}$$

The radiative heat flux in terms of Rosseland approximation is [26]:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial r}=-\frac{16\sigma^*}{3k^*}T^3\frac{\partial T}{\partial r}. \tag{7}$$

In above expression $k^*$ represents the mean absorption coefficient and $\sigma^*$ the Stefan-Boltzman constant.

Considering

$$\left.\begin{array}{c}u=axf'(\eta),\ v=ayg'(\eta),\ w=-\sqrt{a\nu_f}(f(\eta)+g(\eta))\\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\ \eta=\sqrt{\frac{a}{\nu_f}}z\end{array}\right\} \tag{8}$$

Eq. (1) is verified and Eqs. (2)–(5) take the form

$$\frac{1}{(1-\phi)^{2.5}\left(1-\phi+\frac{\rho_{CNT}}{\rho_f}\phi\right)}(f'''-\omega f')+f''(f+g)-(1+F_I)f'^2=0, \tag{9}$$

$$\frac{1}{(1-\phi)^{2.5}\left(1-\phi+\frac{\rho_{CNT}}{\rho_f}\phi\right)}(g'''-\omega g')+g''(f+g)-(1+F_I)g'^2=0, \tag{10}$$

$$\left.\begin{array}{l}\frac{1}{\left(1-\phi+\frac{(\rho c_p)_{CNT}}{(\rho c_p)_f}\phi\right)}\left[\frac{k_{nf}}{k_f}\theta''+\lambda\,\mathrm{Pr}\,\theta+Nr(\theta(\theta_w-1)+1)^2\right.\\ \left.\times\left\{\begin{array}{c}3\theta'^2(\theta_w-1)\\ +\theta''(\theta(\theta_w-1)+1)\end{array}\right\}\right]+\mathrm{Pr}\,\theta'(f+g)=0,\end{array}\right\} \tag{11}$$