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On radiative heat transfer in stagnation point flow of MHD Carreau fluid over a stretched surface

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ABSTRACT

This paper investigates the behavior of MHD stagnation point flow of Carreau fluid in the presence of infinite shear rate viscosity. Additionally heat transfer analysis in the existence of non-linear radiation with convective boundary condition is performed. Moreover effects of Joule heating is observed and mathematical analysis is presented in the presence of viscous dissipation. The suitable transformations are employed to alter the leading partial differential equations to a set of ordinary differential equations. The subsequent non-straight common ordinary differential equations are solved numerically by an effective numerical approach specifically Runge-Kutta Fehlberg method alongside shooting technique. It is found that the higher values of Hartmann number (M) correspond to thickening of the thermal and thinning of momentum boundary layer thickness. The analysis further reveals that the fluid velocity is diminished by increasing the viscosity ratio parameter (β^*) and opposite trend is observed for temperature profile for both hydrodynamic and hydromagnetic flows. In addition the momentum boundary layer thickness is increased with velocity ratio parameter (α) and opposite is true for thermal boundary layer thickness.

Introduction

Now a days flows of non-Newtonian liquids in the presence of magnetic field have significant role in a number of industrial and engineering processes. The common examples of such magneto fluids include plasmas, salt water and electrolytes. The basic concept behind magnetohydrodynamics is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself. The pioneer work on MHD flow past a stretching surface was done by Palov [1]. After that Andersson [2] inspected the MHD flow of a viscous fluid. Moreover Makinde et al. [3] discovered the MHD variable viscosity flow over a convectively heated plate in porous medium along thermophoresis and radiative heat transfer. Few latest studies in this direction can be seen through the attempts [4–6]. Sakiadis [7] discussed the boundary layer behavior on a moving surface and he applied similarity transformations to the boundary layer equations and then numerically solved. Crane [8] simplified the work of Sakiadis.

Thermal radiation is one of the key components of heat exchange. It is produced by the thermal motion of charged particles in matter. All matter with a temperature greater than absolute zero emits thermal radiation. Heat transfer analysis with radiation plays an important role in industrial and technological process. This contains the design of furnace, heat exchangers, safety of nuclear reactor, power plants and turbid water bodies [9]. Various discoveries have been accounted on the boundary layer flows in the stagnation point region. Stagnation points have huge applications in real world and mechanical procedures. These procedures incorporate blowing glass, drying and cooling of papers and other mechanical procedures in designing. The steady two dimensional flow with stagnation point in an incompressible micro polar fluid over a stretching sheet has been studied by Nazar et al. [10]. Farooq et al. [11] studied the stagnation point flow with MHD in a viscoelastic nano fluid with non-linear radiation effects. Heat transfer with porous medium over a stretching sheet with thermal radiation and variable thermal conductivity was discussed by Cortell [12]. Moreover, a numerical examination of heat transfer and flow of Carreau fluid in cylindrical coordinates was discovered by Khellaf and Lauriat [13]. Effect of Carreau fluid flow down an inclined plane with a free surface was inspected by Tshehla [14]. Abbasi et al. [15] discovered the MHD peristaltic transport of Carreau fluid in curved channel with Hall effects.

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Further, the impact of thermal radiation is important in space innovation and high temperature forms. Hossain et al. [16] explored the thermal radiation's effects with the Rosseland diffusion approximation on convective flow over a vertical uniformly heated porous plate. Later on, Hayat et al. [17] inspected the MHD three dimensional flow of a nano fluid with nonlinear thermal radiation and velocity slip. Also Hayat et al. [18] discussed the Oldroyd-B nanofluid flow with MHD over a stretching sheet with heat generation/absorption. Recently Khan and Hashim [19] explored the MHD flow with stagnation point and heat transfer in Carreau fluid along with convective boundary conditions. Additionally, Khan et al. [20] investigated the Carreau fluid with MHD over a convectively heated surface with nonlinear radiation. Advancements in the study of non-Newtonian fluids have been made by different authors [27–31].

The aim of the present study is to address the effects of the MHD Carreau fluid in stagnation point flow with infinite shear rate viscosity. Additionally, Joule heating and nonlinear radiative heat transfer is studied in the presence of convective boundary condition. It is important to note that Carreau fluid is a distinct class of generalized Newtonian fluid which classifies shear thinning and shear thickening nature of fluids. The governing partial differential equations are converted to a set of non-linear ordinary differential equations. Then are solved numerically by applying Runge-Kutta fourth-fifth order method via shooting technique. Current research graphically presents the physical importance of the parameters on the temperature and velocity profiles. The influences of the pertinent flow variables M, α, β^*, N_R, θ_w and γ are described through tables and graphs.

Mathematical formulation

We examine the steady boundary layer flow of an incompressible Carreau viscosity liquid model in the region of stagnation point near a stretching surface. The flow is initiated by a linear stretching surface. The coordinate system is designated in such a way that x-axis is measured alongside the stretching sheet while y-axis is normal to it and fluid conquers the space $y > 0$. The magnetic field B_0 is uniform and applied in y direction and the induced magnetic field is neglected under low magnetic Reynolds number assumption. The sheet velocity is assumed to be $u_w(x) = cx$ with $c > 0$ is stretching rate. The velocity of exterior flow is $u_\infty = ax$ $(a > 0)$, where a is constant. Moreover, heat transfer analysis is completed along the nonlinear thermal radiation with convective boundary condition at the surface. The viscous dissipation and Joul heating effects are also incorporated (Fig. 1).

The constitutive equations for the generalized Newtonian Carreau fluid [20,21] are given as

$$\boldsymbol{\tau} = -p\mathbf{I} + \mu(\dot{\gamma})\mathbf{A}_1, \quad \mu = \mu_0\left[\beta^* + (1-\beta^*)[1+(\Gamma\dot{\gamma})^2]^{\frac{n-1}{2}}\right]. \tag{1}$$

Here $\boldsymbol{\tau}$ is the Cauchy stress tensor, p the pressure, $\mathbf{A}_1$ the first Rivlin-Erickson tensor, $\mathbf{I}$ the identity tensor, $\dot{\gamma} = \sqrt{\frac{1}{2}\Pi}$ with Π as the second invariant strain tensor and defined as $\Pi = trace(\mathbf{A}_1^2)$, n the power law index, Γ a material time constant and $\beta^* = (\mu_\infty/\mu_o)$ the viscosity ratio parameter with μ_0 the zero shear rate viscosity, μ_∞ the infinite shear rate viscosity and taken to be less than one here.

Under the above assumptions and the usual boundary-layer approximations, the governing boundary layer equations for present flow are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

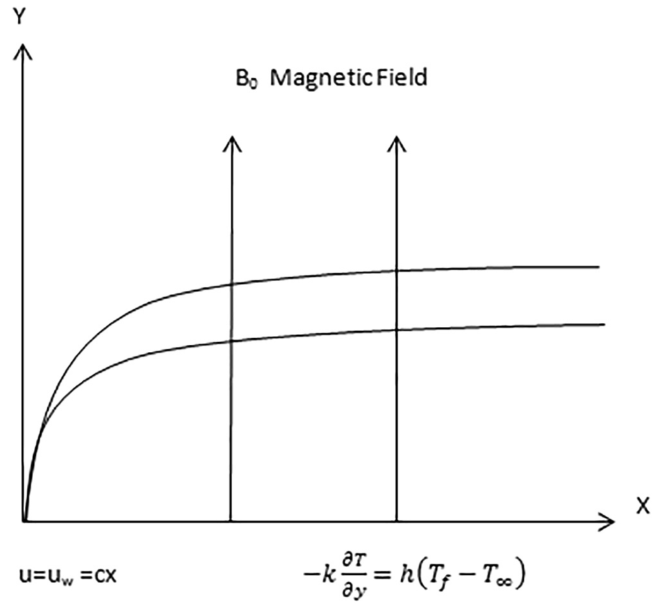

Fig. 1. Physical model under consideration.

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_\infty\frac{du_\infty}{dx} &+ \nu\left(\frac{\partial^2 u}{\partial y^2}\right)\left[\beta^* + (1-\beta^*)\left\{1+\Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\right\}^{\frac{n-1}{2}}\right] \\ &+ \nu(n-1)(1-\beta^*)\Gamma^2\left(\frac{\partial^2 u}{\partial y^2}\right)\left(\frac{\partial u}{\partial y}\right)^2\left\{1+\Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\right\}^{\frac{n-3}{2}} \\ &+ \frac{\sigma B_0^2}{\rho}(u_\infty - u), \end{aligned} \tag{3}$$

$$\begin{aligned} u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_1\frac{\partial^2 T}{\partial y^2} &- \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{\sigma B_0^2}{\rho c_p}u^2 \\ &+ \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2\left[\beta^* + (1-\beta^*)\left\{1+\Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\right\}^{\frac{n-1}{2}}\right]. \end{aligned} \tag{4}$$

In the above equations, $\alpha_1 = \frac{k}{\rho c_p}$ is the thermal diffusivity with c_p the specific heat and k the thermal conductivity, σ the electrical conductivity of the fluid and $\nu = \frac{\mu_0}{\rho}$ the kinematic viscosity of the base fluid.

Note that fluid is portrayed as Newtonian fluid for $n = 1$ and/or $\Gamma = 0$, shear thinning for $0 < n < 1$ and shear thickening for $n > 1$.

Radiative heat flux used in Eq. (4) is given by the Roseland approximation [22]

$$q_r = -\left(\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}\right), \tag{5}$$

where σ^* and k^* are the Stefan-Boltzman constant and the mean absorption coefficient, respectively. For a planer boundary layer flow over a heated surface, Eq. (5) can be written as [23]

$$q_r = -\frac{16\sigma^*}{3k^*}\left(T^3\frac{\partial T}{\partial y}\right). \tag{6}$$

Using Eq. (6) the energy Eq. (4) can be composed as

$$\begin{aligned} u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}&\left[\left(\alpha + \frac{16\sigma^*}{3k^*}\frac{T^3}{\rho c_p}\right)\frac{\partial T}{\partial y}\right] + \frac{\sigma B_0^2}{\rho c_p}u^2 \\ &+ \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2\left[\beta^* + (1-\beta^*)\left\{1+\Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\right\}^{\frac{n-1}{2}}\right]. \end{aligned} \tag{7}$$