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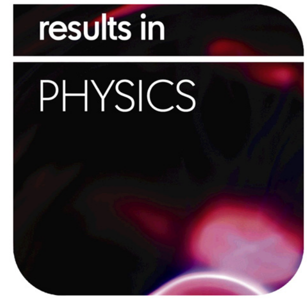

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New integrable problems in a rigid body dynamics with cubic integral in velocities.

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ABSTRACT

We introduce a new family of the 2D integrable mechanical system possessing an additional integral of the third degree in velocities. This system contains 20 arbitrary parameters. We also clarify that the majority of the previous systems with a cubic integral can be reconstructed from it as a special version for certain values of those parameters. The applications of this system are extended to include the problem of motion of a particle and rigid body about its fixed point. We announce new integrable problems describing the motion of a particle in the plane, pseudosphere, and surfaces of variable curvature. We also present a new integrable problem in a rigid body dynamics and this problem generalizes some of the previous results for Sokolov-Tsiganov, Yehia, Stretensky, and Goriachev.

1. Introduction

From about 150 years or so, Bertrand was interested in searching for the structure of forces acting on the motion of a particle in the Euclidean plane which guarantees the existence of an additional integral of a certain form. His accomplishment was slightly in solving this problem for simple forms of the integral such as a polynomial up to the third degree and a fractional function in which the numerator and denominator are linear in velocity variables [1, 2]. He was followed by Darboux who studied the construction of an integrable system having a quadratic integral and solved the problem completely [3](also, see[4]). This direction of research was studied by several authors and it is called in literature *a direct method for obtaining the second invariant*. The majority of integrable mechanical systems describing the motion of a particle in the plane having a polynomial additional integral in the velocities with degree ranging up to six were collected in Hietarienta's review [5]. Other systems were mainly presented by trials to insert new arbitrary parameters to the structure of previous knowing results [6] or by some changes in the methodology [7]. Other types of integrable systems which have a configuration space involving a large numbers of arbitrary parameters were introduced and they were classified and interpreted physically by Gaussian curvature. For instance, when Gaussian curvature vanishes (equals a negative value), the configuration space becomes Euclidean plane (Pseudosphere) (see, e.g.[8]).

The model of a rigid body acts a good example in the integrability problems owing to its applications in diverse branches of science such as astronomy and physics (see, e.g., [9, 10, 11]). Integrable problems concerning the rigid body dynamics and their generalizations to a gyrostat were classified into general and conditional integrable problems according to their validity on an arbitrary level of a cyclic integral or on a fixed level (usually zero-level) of it. The general integrable problems were tabulated in small tables (see, e.g., [12, 13, 14]) and some other problems were added (see, for example, [15]). The famous integrable problems bearing the names of Goriachev- Chaplygin [12] are the first example of conditional integrable problems and they were followed by numerous generalizations (see, for example, [16, 17, 18]).

It is worthly notice that the first integrable mechanical sys-

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