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Effects of heat and mass transfer on unsteady boundary layer flow of a chemical reacting Casson fluid

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ABSTRACT

In this study, an endeavor is to observe the unsteady two-dimensional boundary layer flow with heat and mass transfer behavior of Casson fluid past a stretching sheet in presence of wall mass transfer by ignoring the effects of viscous dissipation. Chemical reaction of linear order is also invoked here. Similarity transformation have been applied to reduce the governing equations of momentum, energy and mass into non-linear ordinary differential equations; then Homotopy analysis method (HAM) is applied to solve these equations. Numerical work is done carefully with a well-known software MATHEMATICA for the examination of non-dimensional velocity, temperature, and concentration profiles, and then results are presented graphically. The skin friction (viscous drag), local Nusselt number (rate of heat transfer) and Sherwood number (rate of mass transfer) are discussed and presented in tabular form for several factors which are monitoring the flow model.

Introduction

The study of heat and mass transfer effects has a lot of applications in engineering especially in industry and manufacturing processes. For example, extrusion of polymers, copper wires drawing, continuous metals casting, glass-fiber production, human transpiration, atomic power plants, cooling of electronic equipment, filtration, refrigeration, spreading of chemical pollutants in plants, injection and diffusion of medicine in blood veins and crude oil's purification. The fluid whose properties cannot be explained by Newtonian fluid models is called a non-Newtonian fluid. Blood cells is a type of non-Newtonian fluid and can be considered as Casson fluid due to the chain structure of blood cells and the substances like fibrinogen, rouleaux, protein etc. There are many other important and strong applications of Casson fluids for example, in industry; fluids behave like elastic solids and for such fluids, a yield shear stress exists in the constitutive equations. Recently time dependent/independent boundary layer models of Casson fluid has attained phenomenal attention due to its rheological applications especially in chemical and mechanical engineering. Researchers, numerical analyst and engineers which are attached with that area of research are putting their efforts to solve these complex Casson fluid models [1–8]. A stretched medium is a kind of sheet that deals with the ambient fluid both thermally and mechanically during a manufacturing process. That's the reason, the fluid flow behavior past that surface, which involves in finding the rate of cooling, has great importance in industrial, manufacturing and technological processes like polymer films or thin sheets production [9–11]. Crane [12] was the first who work on the fluid's flow of stretching sheet of linear order in 1970 and find the similarity solution of the steady-problem. Chiam [13,14] also work on stagnation point flow past a stretching sheet in 1994 where velocity of stretching sheet is equal to the straining velocity of stagnation point flow, then extended the idea to heat transfer with variable conductivity past a stretching sheet in 1996. Some of the research work related to stagnation point flow over a stretching/shrinking sheet to above one is mentioned in [15–23].

Due to amicable applications of stretching plates and a non-Newtonian fluid like Casson fluid, attracts many scientist and researchers. K Bhattacharyya do work by adding heat transfer and magnetic effects in the model of Casson fluid past a stretching sheet [24]. Already dual solution in boundary layer flow with mass transfer analysis have been obtained by Bhattacharyya et al. [25] and extended it to obtaining the analytic solutions of MHD Casson fluid flow over stretching/shrinking sheet with suction or injection effects [26]. Shehzad and Hayat [27] find the series solution after analyzing the non-linear steady model under mass transfer effects on MHD Casson fluid model with chain reaction and suction effects; where similar effects are seen under the influence of

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Nomenclature

u	axial velocity part along x-axis	D	diffusion coefficient
v	transverse velocity part along y-axis	ψ	physical stream function
x	horizontal coordinate	β	Casson parameter
y	vertical coordinate	a	straining rate parameter
ρ	density of fluid	b	stretching rate parameter
υ	kinematic viscosity	γ	velocity ratio parameter
μ	dynamic viscosity	η	similarity variable
μ_D	plastic dynamic viscosity	ϑ	dimensionless stream function
Y_δ	fluid's yield stress	A	unsteady parameter
U_w	velocity of the stretching surface	ϑ	dimensionless temperature
V_w	wall mass suction/injection	Φ	dimensionless concentration
U_∞	straining velocity	f_0	wall mass transfer parameter
τ	temperature of the field	β^*	reaction rate parameter
$\mathbb{C}$	species concentration	S_C	Schmidt number
τ_w	temperature near to sheet	Pr	Prandtl number
τ_∞	temperature away from sheet	C_f	Skin friction coefficient
$\mathbb{C}_w$	constant concentration	N_u	Nusselt number
$\mathbb{C}_\infty$	concentration in free stream	S_h	Sherwood number
R	reaction rate of solute		

magnetic and Casson parameter on the velocity profile. Sandeep [28] present work with the collaboration of other researchers in finding the analytical solutions of Casson fluid flow past a stretchy sheet which is permeable and exponentially long where dual results are obtained and shows the comparison between Newtonian and Casson fluid. Recently, Bilal and Hayat [29] worked on steady model of MHD mixed convection Casson fluid flow with the involvement of Hall and thermal diffusion effects past a stretching sheet. Most of the models of Casson fluid models in heat and mass transfer analysis are steady. Unsteady models of non-Newtonian fluids past a stretching sheet have gained less attentions. However, the unsteady flow models with irregular domains are also under interest as Dehghan [30] work on time dependent incompressible Navier-Stokes equations by introducing some new numerical techniques. Recently, he [31,32] shows tremendous work in boundary layer problems containing irregular domain and provides the numerical plan for 2D Rayleigh-Stokes model with fractional derivative. Also, Tsai [33] give the solutions of highly nonlinear partial differential equations with irregular domain by using hybrid homotopy technique (HAM; homotopy analysis method + MFS; method of fundamental solutions + APS; Augmented polynomial spline). In present work, HAM is provoked to get the solution of an unsteady Casson fluid model over simple domain past a stretching sheet with heat, mass transfer along 1st order chemical reaction.

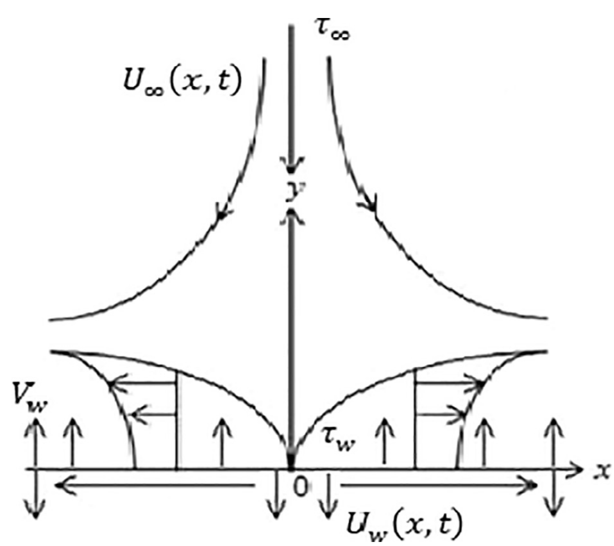

Fig. 1. Flow Model.

Flow analysis

Consider the unsteady two-dimensional stagnation point flow of a non-Newtonian Casson fluid over a stretching sheet. The fluid flow is restricted to $y > 0$ with the involvement of 1st order chemical reaction. Fig. 1 tells that flow is modelled by stretching of a bounding and non-conducting sheet. The wall is stretched by applying two equal and opposite forces along the x-axis, keeping the origin fixed in such a way that the rate of movement of the sheet is of 1st order in that flow regime. For an isotropic and incompressible Casson fluid flow, the rheological equation of state can be stated as (see [34])

$$\tau_{ij} = \begin{cases} \left(2\mu_D + Y_\delta\sqrt{\frac{2}{\pi}}\right)e_{ij}, & \pi > \pi_p \\ \left(2\mu_D + Y_\delta\sqrt{\frac{2}{\pi_p}}\right)e_{ij}, & \pi < \pi_p \end{cases}$$

where μ_D is the plastic dynamic viscosity, Y_δ is the fluid's yield stress, $\pi = e_{ij}e_{ij}$ is the multiplication of the component of deformation rate with itself, e_{ij} is the $(i,j)^{th}$ component of the deformation rate and π_p is the critical value of this product based on that model. The governing equations for above flow model are (see details [35]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial U_\infty}{\partial t} + U_\infty\frac{\partial U_\infty}{\partial x} + v\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$\frac{\partial \tau}{\partial t} + u\frac{\partial \tau}{\partial x} + v\frac{\partial \tau}{\partial y} = \alpha\frac{\partial^2 \tau}{\partial y^2} \quad (3)$$

$$\frac{\partial \mathbb{C}}{\partial t} + u\frac{\partial \mathbb{C}}{\partial x} + v\frac{\partial \mathbb{C}}{\partial y} = D\frac{\partial^2 \mathbb{C}}{\partial y^2} - R(\mathbb{C} - \mathbb{C}_\infty) \quad (4)$$