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Corrections for the combined effects of decay and dead time in live-timed counting of short-lived radionuclides

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HIGHLIGHTS

- Derived combined effects of decay and dead time.
- Derived for counting systems with extending or non-extending dead times.
- Derived series expansions for both midpoint and decay-to-start-time methods.
- Useful for counting experiments with short-lived radionuclides.
- Examples given for ^{15}O, used in PET scanning.

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ABSTRACT

Studies and calibrations of short-lived radionuclides, for example ^{15}O, are of particular interest in nuclear medicine. Yet counting experiments on such species are vulnerable to an error due to the combined effect of decay and dead time. Separate decay corrections and dead-time corrections do not account for this issue. Usually counting data are decay-corrected to the start time of the count period, or else instead of correcting the count rate, the mid-time of the measurement is used as the reference time. Correction factors are derived for both those methods, considering both extending and non-extending dead time. Series approximations are derived here and the accuracy of those approximations are discussed.

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1. Introduction

Studies and calibrations of short-lived radionuclides are of particular interest in nuclear medicine. For instance, positron emission tomography (PET) imaging by ^{15}O, with a half-life of 122.46 s (Chisté and Bé, 2015), is the "gold standard" for assessment of blood flow in studies of tumor vascularity (Aboagye et al., 2012).

Yet, counting measurements on short-lived nuclear states (either excited states or radionuclides) using live-timed counting systems are vulnerable to an error due to the combined effect of decay and dead time. The effect causes a bias in the apparent count rate due to the changing live-time fraction throughout the measurement. Since the count rate is decreasing during the measurement, the fractional live-time is increasing and the latter parts of the measurement contribute relatively more to the recorded count rate. If decay corrections do not account for this, then there remains a bias in the count rate at the reference time, which could be important when producing calibration sources or standards for short-lived radionuclides.

Counting experiments are subject to dead time, τ, which is the period of time following an event during which any part of the apparatus is unable to count another event. Moreover, a dead time is *extending* if events that are lost during that time cause further dead time, or *non-extending* if lost events do not cause dead time. A system could have *mixed* dead time, which is not considered further here, beyond noting that by designing a counting system with an imposed, extending dead-time that is longer than any intrinsic dead time of the system, one can approximate the pure extending dead-time case. The uncertainties from various dead-time, pileup, and live-time corrections are the subject of a recent review article (Pommé et al., 2015), which indeed cites the final result of this paper.

Axton and Ryves (1963) derived an approximate correction formula with leading-order expansion for the count rate at the start of the counting period for non-extending dead times. Müller (1981) derived expressions for non-extending and extending systems without live-timing. For modern live-timed systems, which may have imposed dead times, it is desirable to use the live-timed

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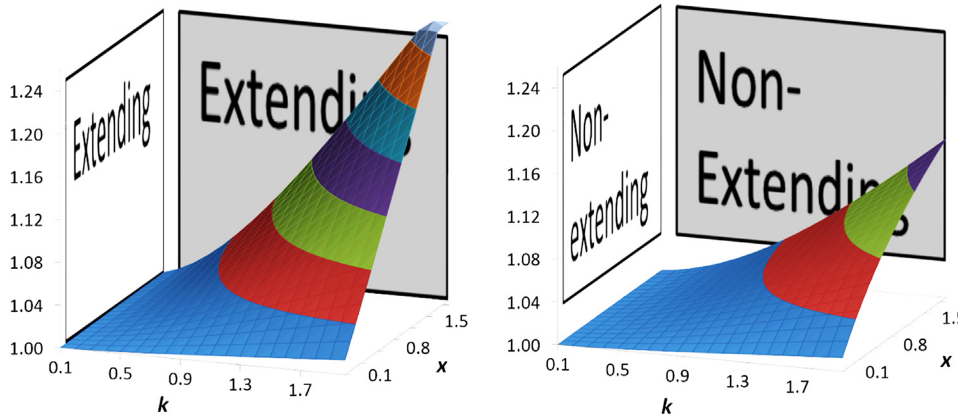

Fig. 1. Plots of f_d as a function of k and x for extending (left) and non-extending (right) dead-times.

count rate determined by the instrument, rather than calculating the dead-time correction.

Derivations are presented for the actual and apparent live count rate for a decaying source over a finite measurement duration. Furthermore, correction factors are defined for both the decay-to-start method and midpoint approximation. These expressions are meant to enable a simple implementation of the corrections and aid in experimental design, thereby limiting errors caused by this combination dead-time and decay effect. Tabulated results are presented for various conditions and as well as simple approximations to the correction formulas.

Finally, the equivalence between the correction factors derived in this work and the approximations given in previous reports is demonstrated for certain cases.

2. Derivation

From the start, let us note that the background count rate, B, will be ignored in this work because it would hardly affect the final correction factors f_m and f_d that conclude this paper. This fact was verified by deriving the results with B included and noting the equivalence to the present results for extending dead time. For the non-extending case, the change in the correction factor due to B for initial net count rate due to the radionuclide, $\rho(0)$, is of the order $B/\rho(0)$. As the corrections derived here are needed only at high count rates, this small correction on the correction is not significant.

Consider a counting experiment with initial true, count rate at time $t=0$ of $\rho(0)$. If the measurement is live-timed with an extending or non-extending dead time, τ, then the observed average live-timed count rate, $\bar{R}$, is the ratio of the total live counts, N_L, to the total live time, T_L. That is,

$$\bar{R} = \frac{N_L}{T_L} \tag{1}$$

When $\rho(t)$ is decreasing exponentially with decay constant, λ, as,

$$\rho(t) = \rho(0)e^{-\lambda t} \tag{2}$$

then one of two methods is typically used to report the experimental value for the instantaneous true count rate from the observed average rate. For a total count time, T_{tot}, that is much lower than λ^{-1}, the decay is nearly linear during the measurement. In that case, the midpoint method is sometimes employed. That is, $\bar{R}$ is assumed to approximate the true rate at the mid-time,

$$\rho\left(\tfrac{1}{2}T_{tot}\right) \approx \bar{R} \tag{3}$$

A more accurate method is to average Eq. (2) over the domain from $t=0$ to $t=T_{tot}$ and then solve for $\rho(0)$ in terms of the average rate, $\bar{\rho}$, as

$$\rho(0) = \bar{\rho}\ \frac{\lambda T_{tot}}{1 - e^{-\lambda T_{tot}}} \tag{4}$$

Ignoring the decay-dead-time effect in this case is equivalent to the assumption that,

$$\rho(0) \approx \bar{R}\ \frac{\lambda T_{tot}}{1 - e^{-\lambda T_{tot}}} \tag{5}$$

We next calculate $\bar{R}$ from $\rho(t)$ and the resulting correction factors for the two assumptions.

For a counting system with a fixed, extending or non-extending dead time, τ, the instantaneous recorded count rate, $R'(t)$, is given by the well-known formulae (ICRU, 1994):

Extending:

$$R'(t) = \rho(t)\, e^{-\rho(t)\tau} \tag{6a}$$

Non-extending:

$$R'(t) = \frac{\rho(t)}{1 + \rho(t)\,\tau} \tag{6b}$$

For measurement duration T_{tot}, the recorded live counts N_L and live time T_L can be calculated as:

$$N_L = \int_0^{T_{tot}} R'(t)\ dt \tag{7}$$

Extending:

$$T_L = \int_0^{T_{tot}} e^{-\rho(t)\tau}\ dt \tag{8a}$$

Non-extending:

$$T_L = \int_0^{T_{tot}} \frac{1}{1 + \rho(t)\ \tau}\ dt \tag{8b}$$

The calculated value of $\bar{R}$, called $\bar{R}_c$, can now be written explicitly by combining Eqs. (2), (6)–(8) as:

Extending:

$$\bar{R}_c = \frac{\int_0^{T_{tot}} \rho(0)e^{-\lambda t}e^{-\left(\rho(0)e^{-\lambda t}\right)\tau}dt}{\int_0^{T_{tot}} e^{-\left(\rho(0)e^{-\lambda t}\right)\tau}dt} \tag{9a}$$