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The Linear Matching Method applied to composite materials: A micromechanical approach

O. Barrera [a], A.C.F. Cocks [b], A.R.S. Ponter [c,*]

[a] Department DASTEC University "Mediterranea" of Reggio Calabria, Via Melissari Feo di Vito, I-89124 Reggio Calabria, Italy
[b] Department of Engineering Science University of Oxford, Parks Road, Oxford OX1 3PJ, UK
[c] Department of Engineering University of Leicester, Leicester LEI 7RH, UK

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The paper considers a Direct Method for the evaluation of the maximum load corresponding to pre-assigned limits on the non-linear behaviour of the matrix and fibres in a laminate structure. This is achieved by combining a consistent micro–macro model for linear behaviour with an extension of the Linear Matching Method (LMM), previously extensively applied to Direct Methods in plasticity. The method is developed with assumptions that allow the methodology to be displayed in its simplest form. Applications to examples of laminate elements and a laminate plate containing a hole are described, assuming a matrix with a limit on ductility.

1. Introduction

This paper is concerned with methods for assessing the load bearing capacity of composite laminate structures, corresponding to appropriately defined micro failure criteria. The conventional approach to the assessment of the maximum load on a structure first requires the formulation of constitutive equations for a material element, incorporating an understanding of deformation and failure modes within the composite microstructure. This is then followed by a conventional step by step analysis to failure. The relationship between the critical micro structural failure mechanism and the allowable maximum load is understood through the structure of the constitutive relationships and detail can become lost. The work of this paper falls within the general field of Direct Methods [1], by which we mean computational or analytic methods that allow the direct evaluation of the load or load range corresponding to a pre-assigned material or design constraint, such as a yield condition or a critical strain. In the case of laminates such Direct Methods may be applied to a constitutive model (see [22,23]) although the behaviour of laminates may not be well described by elastic-perfect plasticity. Here we explore the possibility of developing a method whereby the maximum permissible load may be related directly to a particular strain condition or failure mode at any point in the structure, in any layer of the laminate and at any point in the microstructure, taking into account non-linear behaviour of the composite material constituents. Such a proposal requires a degree of complexity which we wish to avoid in this first attempt. We combine a kinematically consistent micromechanics model with significant elements of classical laminate theory [25] and a simple description of micro failure. This first step then allows the methodology of such a Direct Method to be displayed in its simplest form.

The history of the development of laminate failure criteria is well known. Since the work of Hill [2,3] and Tsai–Wu [4] a range of empirical and micromechanical failure criteria have been proposed which mathematically combine lamina mechanical properties into an assumed homogeneous laminate to attain idealized uniform strength and stiffness throughout the structure. Failure theories, based on a micromechanics approach, were first developed by Hashin and Rotem [5,6]. Their method can be viewed as a macro approach based on micro mechanical issues that consider a failure criterion on the basis of observed failure modes. Another step forward was made by Chang et al. [7,8] by introducing other possibilities of failure: matrix cracking, fibre–matrix interface shearing, fibre breakage and material property degradation. Other researchers [9–13] treated the failure of the matrix and of the fibre separately, making some simplified assumptions. These authors employ a Mohr–Coulomb yield criterion for the matrix and assume that final failure is determined by the failure of the fibres. Mayes and Hansen [14] introduced a theory in which micro-level stresses, evaluated considering constitutive behaviour of the components, are applied to different failure theories for both the fibres and

* Corresponding author. Address: Flat 4, 20, Great Pulteney Street, Bath BA2 4BT, UK. Tel.: +44 122533430.
E-mail addresses: olga.barrera@unirc.it (O. Barrera), alan.cocks@eng.ox.ac.uk (A.C.F. Cocks), asp@le.ac.uk (A.R.S. Ponter).

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matrix. This approach includes the effect of changes in the constitutive properties of the components. For instance, similar methods were used to predict non-linear stress–strain relations and failure modes for metal and polymeric matrix composites. Since then a number of developments have taken place and the subject remains one of active research. It should be emphasised that the work in this paper is not concerned with the micromechanics of failure, rather the development of the structure of computational method that is capable of taking such micro failure mechanisms into account and directly relating them to the maximum load bearing capacity of a structure. Keeping in mind the needs to first display a simple theory, we restrict attention to non-linear behaviour of a ductile matrix, assuming failure occurs at a critical micro strain. This is described in detail in Section 3. To develop a complete method, capable of comparison with experimental data, it is necessary to incorporate into the theoretical structure described here a set of possible failure modes of both matrix and fibres and this awaits further development. This is discussed further in Section 6.

The Linear Matching Method (LMM), the method applied here, consists, essentially, of a programming method that allows a direct evaluation of the load corresponding to predefined kinematic restraints [15–21], following an iterative process. The primary area of application has been the evaluation of limits appropriate for metallic structures subjected to severe thermo-mechanical loads. Methods have been developed for the life assessment of power plant at high temperature and an entire set of such methods now form part of the UK Life Assessment Method R5 [17,18]. The work in this paper is a first attempt to develop an appropriate method of this type for composite materials.

The elements are as follows. A structure is subjected to a load distribution where F_i is a chosen load distribution on S_F, part of the surface S, and λ is a scalar multiplier. The objective of the method is to evaluate the value of λ so that the strain field in the microstructure corresponds to a prescribed design condition, while all other conditions of the continuum problem are satisfied: equilibrium, compatibility and consistency with the material behaviour. LMM is applicable to material behaviour where a convex strain potential exists, implying that a deformation theory of inelastic behaviour must be adopted. A summary of the theoretical basis of the method, as required for this application, is given in the Appendix. For the case discussed here, the material behaviour is assumed to be either linearly elastic or elastic-perfectly-plastic, without unloading. The method is an iterative process where each iteration contains four stages. The process begins with the generation of a kinematically admissible strain field, conventionally by solving a linearly elastic problem for an arbitrary λ. This strain field is then scaled so that the design constraint is satisfied. The process then consists of the following stages;

Stage 1: A matching linear material is defined by choosing linear moduli (which are spatially varying) so that the linear material and the actual material give the same stress state for this initial strain field. The solution of the resulting linear problem produces a new strain field which reduced the potential energy of the structure. This is discussed in detail in the Appendix.
Stage 2: The new strain field will generally not satisfy the design constraint and is scaled by a factor, to ensure that the design constraint is re-imposed.
Stage 3: The load parameter λ is now changed so that the potential energy is a minimum for the new scaled strain field, amongst all such scaled strain fields.
Stage 4: A lower static load factor is calculated that measures the degree of deviation of the equilibrium stress field in the linear solution from the actual material behaviour. The difference between the load factors evaluated in Stage 3 and Stage 4 indicates the deviation from convergence.

For the material assumptions adopted here, the specific form of this process is discussed in detail in the following sections. A consistent relationship between stresses and strains in microstructure (i.e. the fibres and the matrix) and the ply stresses and strains is described. This allows the design constraint to be assigned to the properties of the materials of the microstructure. Conventional laminate theory then relates the ply stresses to those of the laminate. Although in this application no softening behaviour is allowed, in a recent study such methods have been applied to portal frames with softening elements, with the objective of evaluating the maximum loads associated with differing design limits [20,21]. Hence the method described here is the first step towards evaluation of a maximum load where softening effects occur in the microstructure. Such possibilities are discussed further in Section 6.

The outline of the paper is as follows. The constitutive assumptions of the interaction between the fibre and matrix are discussed in Section 2 and a simplified scheme of the failure modes for continuous fibre composite is reported. In Section 3 the micromechanics model for the evaluation of the stress and the strain at the micro-level is presented. Section 4 concerns the application of the LMM for the evaluation of the peak load where a design failure criteria associated with the fibres and the matrix are taken into account. Numerical applications are reported in Section 5, followed by conclusions in Section 6.

2. Constitutive assumptions of matrix and fibre

Perfect adhesion between fibre and interface is assumed. Interfacial damage (in particular, debonding) and fibre buckling are neglected. We concentrate on two possible failure modes, cracking or strain exhaustion within the matrix, characterised by a maximum effective strain, and the failure of the fibres. The methodology is, however, capable of incorporating a much wider range of possible local failure modes. The local constitutive models for the matrix and fibres are illustrated in Fig. 1 and described in the following.

2.1. Fibre: elastic-brittle behaviour

In the following the superscripts f and m refer to the fibres and the matrix in a layer of a unidirectional composite laminate. The 1 direction is in the direction of the fibre. The fibre reinforcement is considered to be longitudinally continuous, isotropic, elastic and perfectly brittle, with tensile behaviour limited by a maximum principal strain failure criterion. The fibre fails when the maximum longitudinal strain reaches a critical value, i.e.

$$-\varepsilon_f^c \leqslant \varepsilon_1^f \leqslant \varepsilon_f^* \tag{1}$$

where ε_1^f denotes the axial strain in the fibre. ε_f^c and ε_f^* denote failure values in compression and tension.

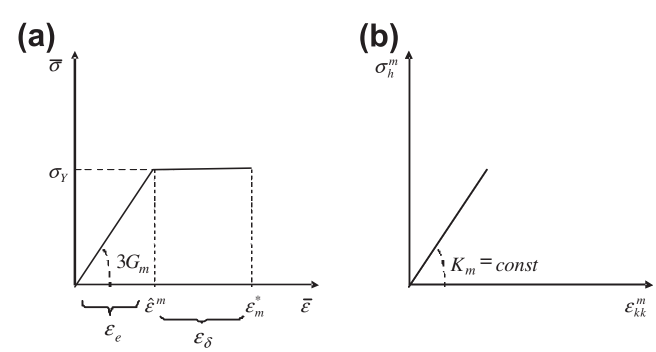

Fig. 1. Constitutive properties of the matrix and fibres.