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Minimization of thermal expansion of symmetric, balanced, angle ply laminates by optimization of fiber path configurations

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ABSTRACT

Optimal fiber path configurations that minimize the sum of the coefficients of thermal expansion (CTE) values along the principal material directions for a class of laminates are presented. Previous studies suggest that balanced, symmetric, angle ply laminates exhibit negative CTE values along the principal directions. Using the sum of the CTE values along the principal material directions as an effective measure of the coefficient of thermal expansion (CTE_{eff}), we have shown and provided a proof that the smallest value of CTE_{eff} is rendered by straight fiber path configurations. The laminates considered are sufficiently thin so as to neglect the thermal stresses induced through the thickness of the laminate. It is found that the minimal CTE_{eff} values occur for $[+45/-45]_{ns}$ lay-ups. This result is supported by numerical studies that consider curvilinear fiber paths. The possibility of obtaining zero CTE values along both principal material directions and the conditions that render this situation are also examined.

1. Introduction

Failure by thermal fatigue can be mitigated by minimizing the coefficient of thermal expansion (CTE) of composite laminates α_X^L and α_Y^L along the principal material directions. Early work on characterization of CTE values in fiber reinforced composites was due to Craft and Christensen [1], Marom and Weinberg [2], Ishikawa and Chou [3], Bowles and Tompkins [4], Sleight [5], Lommens et al. [6], and references therein. More recent studies have focused on laminates with straight fiber configurations [7–10]. Amongst these, those which are balanced, symmetric, angle ply ($[+\theta/-\theta]_{ns}$) lay-ups have been found to exhibit anomalous mechanical response. Analysis of these laminates have shown the existence of negative CTE (α_X^L, α_Y^L) for certain range of ply orientations [7,9]. Zhu and Sun [10], showed that the ratio of shear modulus G_{12} to the Young's modulus E_1 is an important parameter that determines the sign and magnitude of the CTE in the composite laminate. However, negative CTE values along both principal material directions (x, y) of the composite laminate were not obtained simultaneously for any ply angle θ. In this study, we relax the requirement of fibers having straight configurations and seek the optimum fiber path that yields the least value of CTE_{eff}, maintaining the assumption of a balanced, symmetric, angle ply laminate. In the analysis to follow, the possibility of obtaining a zero value for CTE_{eff} is investigated and conditions for obtaining such a CTE_{eff} are derived.

2. Model description

Consider curvilinear fiber configurations in the x–y plane which are symmetric about the z-axis (Fig. 1) in the Representative Unit Cell (RUC) of in-plane dimensions $A \times B$. The fibers are stacked parallel to the y-axis. Obliquely stacked configurations of the fibers are not considered since it reduces to the case under consideration as can be seen from Fig. 2. This would imply that for any infinitesimally small portion of the fiber curve with an orientation θ, there exists an infinitesimally small complementary fiber element with orientation $-\theta$ (Fig. 3). For every fiber at angle $+\theta$ at $z = +z^*$, there is another fiber of same orientation at $z = -z^*$. Also, for every fiber at an angle $-\theta$ at $z = +z^{**}$, there is another fiber at $z = -z^{**}$ with the same orientation. Hence, this configuration acts as a balanced symmetric laminate for which the moment resultants due to thermal stresses cancel out (see [11]), i.e. $M_x^* = 0, M_y^* = 0$ and $M_{xy}^* = 0$. Here, we use standard composite laminate nomenclature as given in [11]. Similarly, the effective shear force resultants due to thermal expansion also cancel out, i.e. $N_{xy}^* = 0$. Therefore, the only non-zero stress resultants present are normal stresses along the principal directions $\left(N_x^* \text{ and } N_y^*\right)$ in the plane of the laminate. The

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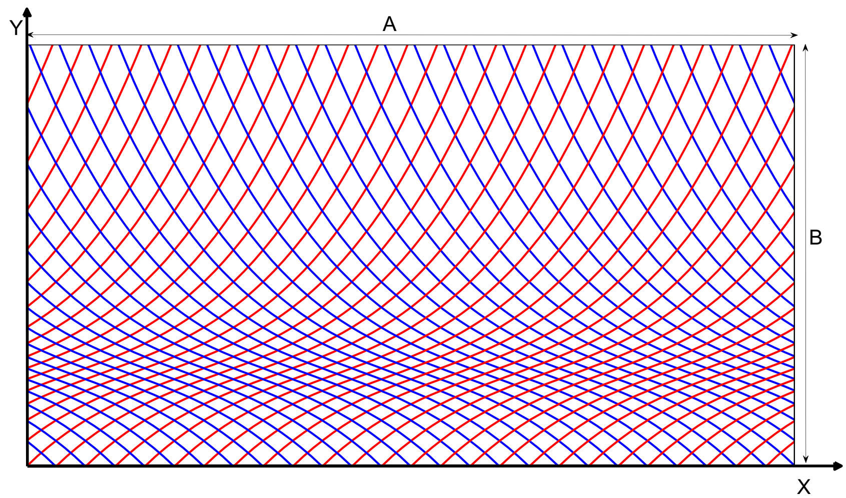

Fig. 1. Profile of fibers in the Representative Unit Cell (RUC) of dimensions $A \times B$. The RUC has many overlaid symmetric fibers which renders the RUC to have a structure similar to that of a balanced, symmetric, angle ply laminate.

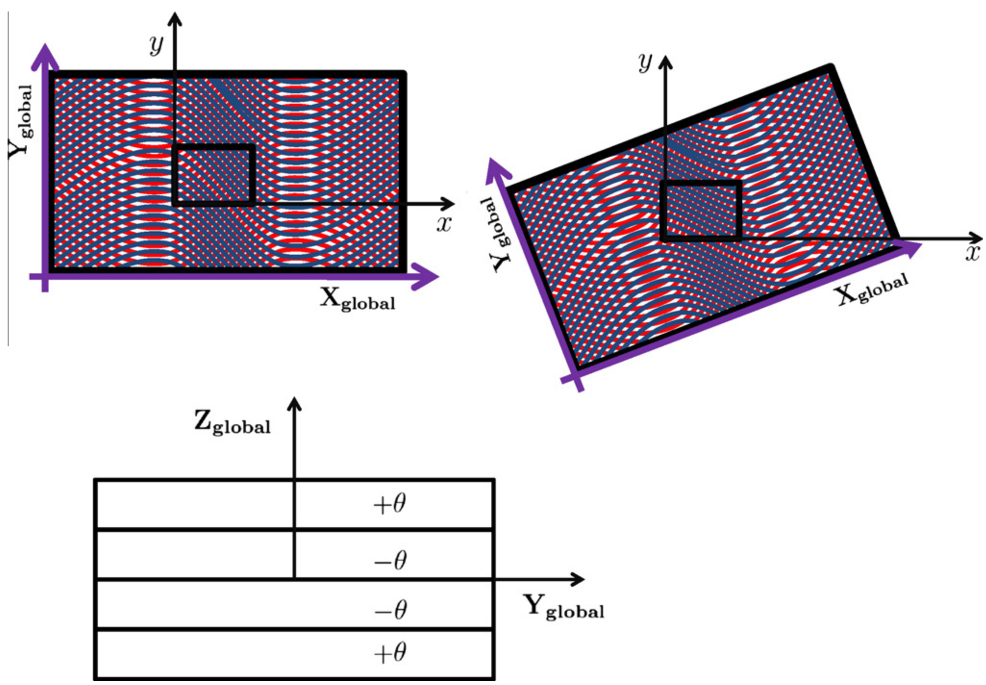

Fig. 2. Schematic showing the equivalence of the fiber stacking along the horizontal and oblique directions.

curvilinear fiber format in the RUC is invariant along the *y*-direction. Hence, the compliance matrix of an infinitesimal strip of width *dx* is a function of *x* alone. The continuity of fiber slopes across adjacent RUCs is ensured by the equality of slope at RUC boundaries.

3. Mathematical formulation

3.1. Straight Fibers

For a straight fiber, balanced, angle ply laminate, if the fiber is oriented at an angle θ with respect to *x*-direction [7,10], we have

$$\alpha_X^L = \alpha_1 \cos^2\theta + \alpha_2 \sin^2\theta + \frac{\bar{S}_{16}}{\bar{S}_{66}}(\alpha_2 - \alpha_1)\sin 2\theta \quad (1)$$

$$\alpha_Y^L = \alpha_1 \sin^2\theta + \alpha_2 \cos^2\theta + \frac{\bar{S}_{26}}{\bar{S}_{66}}(\alpha_2 - \alpha_1)\sin 2\theta \quad (2)$$

where

$$\begin{aligned}\bar{S}_{16} &= \{2(S_{11} - S_{12}) - S_{66}\}\cos^3\theta\sin\theta \\ &\quad + \{2(S_{12} - S_{22}) + S_{66}\}\cos\theta\sin^3\theta \\ \bar{S}_{26} &= \{2(S_{11} - S_{12}) - S_{66}\}\cos\theta\sin^3\theta \\ &\quad + \{2(S_{12} - S_{22}) + S_{66}\}\cos^3\theta\sin\theta \\ \bar{S}_{66} &= 2\{2(S_{11} + S_{22} - 2S_{12}) - S_{66}\}\cos^2\theta\sin^2\theta \\ &\quad + S_{66}\{\cos^4\theta + \sin^4\theta\}\end{aligned} \quad (3)$$

For no thermal expansion, we should have $\alpha_X^L = 0$ and $\alpha_Y^L = 0$ simultaneously. This leads to $(\alpha_X^L - \alpha_Y^L) = 0$.

$$(\alpha_1 - \alpha_2)\left\{\cos 2\theta - \left(\frac{\bar{S}_{16} - \bar{S}_{26}}{\bar{S}_{66}}\right)\sin 2\theta\right\} = 0 \quad (4)$$