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Analytical elastic stiffness model for 3D woven orthogonal interlock composites

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ABSTRACT

This research presents the development of an analytical model to predict the elastic stiffness performance of orthogonal interlock bound 3D woven composites as a consequence of altering the weaving parameters and constituent material types.

The present approach formulates expressions at the micro level with the aim of calculating more representative volume fractions of a group of elements to the layer. The rationale in representing the volume fractions within the unit cell more accurately was to improve the elastic stiffness predictions compared to existing analytical modelling approaches.

The models developed in this work show good agreement between experimental data and improvement on existing predicted values by models published in literature.

1. Introduction

Three-dimensionally (3D) woven composites have been identified as a class of material that have potential performance and manufacturing benefits compared to traditional two-dimensional (2D) laminate composites for structural applications [1–6]. The 3D weaving process controls the placement of reinforcing tows in the *X*, *Y*, and *Z* axis directions. A designer can potentially tailor the performance of the weave architecture to the specific requirements of the application by altering the weaving parameters. There are numerous combinations of weaving parameters that could be selected each of which imparts a different mechanical performance. A lack of understanding currently exists as to the effect on the mechanical performance as a consequence of altering the weaving parameters. To help realise the potential benefits of 3D woven composites, the designer must be facilitated with modelling tools that allows them to quickly evaluate the effect of weaving parameters on the geometric characteristics and mechanical performance.

In literature there are two approaches to facilitate this aim, i.e. Finite Element (FE) and analytical models. The FE approaches have the potential to encapsulate more complexities of the 3D woven composite than analytical methods but are generally too computationally and time intensive [7]. This would make such methods unsuitable when trying to assess quickly numerous permutations of 3D weave architecture and the consequences of altering the constituent materials and weaving parameters on the mechanical performance of the composite. Therefore, a clear need for accurate analytically based approaches is still necessary. There are numerous analytically based models developed to model the mechanical performance of 3D woven composites [8–14]. These analytical approaches use similar principles to formulate relationships based on the spatial orientation of unidirectional tows in the unit cell or a small representative volume of the composite [15,16]. The accuracy of the predicted mechanical properties is only as accurate as the inputted geometric definition/description of the unit cell.

Calculation of the macroscopic properties of the unit cell are dictated by first calculating the properties of the constituent elements and averaging accurately the contribution they make to a macroscopic layer and subsequently the whole unit cell. Various authors analytical approaches accepted highly idealised representation of tow cross-sectional shape. For example Tan et al. [10] presented the *XYZ*, *ZXY* and *ZYX* models to predict the stiffness of 3D woven composites. The representative unit cell was segmented into a number of micro-blocks where the authors proposed a mixed iso-strain and iso-stress scheme to calculate the elastic properties of the 3D woven composite. These micro-blocks could be resin impregnated stuffer, filler or binder tow blocks where the cross-sectional shape of the tow was taken to be rectangular.

Utilising non-representative tow cross-sectional characteristics could lead to inaccurate calculation of volume fraction at the tow element level. This is compounded further when calculating the volume fraction of the respective elements that make up a layer in the unit cell. The highly idealised representation of the geometric characteristics of the constituent parts (stuffers, fillers, binders

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Nomenclature

Abbreviations

2D	two-dimensional
3D	three-dimensional
FE	Finite Element
MOA	Modified Orientation Averaging
OA	Orientation Averaging
T-T-T	Through-The-Thickness

Symbols

A	area (m^2)
AR	aspect ratio of the tow cross-section
C_f	circumference of filler tow (m)
C_{ij}	orthotropic stiffness matrix
E	modulus of elasticity (GPa)
F	float (orthogonal interlock only) number of fillers binder travels over
G	shear modulus (GPa)
h	thickness of a tow (m)
H	thickness of unit cell (m)
l	length a tow (m)
L	length of unit cell (m)
n_s^{uc}	number of stuffers along the weft direction (Y) in the unit cell
n_f^{u}	number of fillers along the warp direction (X) in the unit cell
n_b^{uc}	number of binders along the weft direction (Y) in the unit cell
S_{ij}	orthotropic compliance matrix
T	transformation matrix
V	volume (m^3)
V_m	volume fraction of a matrix
V_o	volume fraction of a tow

Greek letters

ε	strain
ν	Poisson's ration
σ	stress (GPa)

Subscript

1	longitudinal fibre direction (material co-ordinate system)
2	transverse fibre direction (material co-ordinate system)
3	transverse fibre direction (material co-ordinate system)
bv	vertical binder element
bh	horizontal binder element
b	binder tow
e	element
f	filler tow
l	layer
$OUTER$	outer layer in the unit cell
s	stuffer tow
$WARP$	warp layer in the unit cell
$WEFT$	weft layer in the unit cell
x	in the longitudinal x axis
y	in the transverse y axis
z	in the out-of-plane z axis

Superscript

-1	inverse of matrix
e	element
l	layer
T	transpose of matrix

and matrix) that make up a layer must be improved in order to yield better predictions. Existing analytical models [8–11,13,17] present predictions that are generally significantly higher, by 10% compared to the small amount experimental data available in literature.

This paper presents an analytical modelling tool to predict the elastic stiffness properties of 3D woven orthogonal interlock composites. The model assesses the change in performance as a consequence of altering the weaving parameters that dictates the 3D weave architecture. The 3D woven composite modelled in this paper consists of alternate layers of stuffers travelling in the 0° (warp) direction and fillers travelling in the 90° (weft) direction bound Through-The-Thickness in the warp direction by a binding tow (Fig. 1).

Previous work by Buchanan et al. [18] describe the development of a geometric model that is capable of calculating the necessary inputs for the present elastic stiffness model. The geometric model and the present elastic stiffness model are driven by weaving parameters and the constituent material properties from the manufacturer's datasheet. The modelling methodology from the geometric model predicts useful information for the engineer such as areal density, overall thickness and fibre volume fraction in addition to the variables that appear in Eqs. (8)–(16) of this paper. The geometric model also defines the composite unit cell to be modelled by the elastic stiffness model, accepting more representative tow cross-sectional characteristics. The geometric model allowed three representative tow cross-sectional shapes to be used including lenticular, elliptical and racetrack. A racetrack cross section is essentially a rectangle with semi-circles on its ends. Assumption of any of these tow cross-sectional shapes is still an idealisation. For instance Summerscales and Russell [19] found evidence that assuming the lenticular shape to be symmetrical is incorrect. However, one or more of these ideal tow cross sections is often adopted in models [18,20–22] and have shown good agreement between calculated and measured values.

The macroscopic unit cell modelled by the present elastic stiffness model is representative of one repeat of the weave architecture (Fig. 1). The model follows the unit cell discretisation method into layers and elements. Elements within a layer can be pure matrix material, or a combination of reinforcing tows in the X or Y, and Z axis directions. The present approach formulates expressions at the micro level with the aim of calculating more representative volume fractions of a group of elements to the layer. The rationale in representing the volume fractions of elements within a layer and subsequently the layers within the unit cell more accurately was to improve the elastic stiffness predictions compared with existing analytical modelling approaches for example Cox and Dadkhaha [8] and Wu et al. [13].

2. Description of present modelling approach

The modelling approach taken in this paper follows from Naik et al. [11,17] and develops the modelling approach by Wu et al. [13]. The unit cell in this work is representative of one repeat of the weave architecture. The new model treats the 3D woven composite as an assembly consisting of layers containing unidirectional elements (which are fibrous tows encased in resin). The new modelling approach formulates expressions that discretise the unit cell into layers and then elements.