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Interlaminar stresses in composite laminates: Thermoelastic deformation

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ABSTRACT

An approximate elasticity solution for prediction of the displacement, stress and strain fields within the m-layer, symmetric and balanced angle-ply composite laminate of finite-width and subjected to uniform axial extension was developed earlier [4]. In the present paper, the authors have extended that solution to treat thermal stresses and deformations induced by a uniform change in laminate temperature. The results have revealed not only the complex fields within the laminate, but also inter-relationships between the lamina axial and shearing coefficients of thermal expansion and the effective laminate coefficients of thermal expansion. Further, the solution is shown to recover laminated plate theory predictions for thermally induced fields at interior regions of the laminate, thereby confirming the boundary layer nature of the interlaminar phenomena for the thermoelastic case. Finally, the results exhibit the anticipated response in congruence with the mechanical solution of Ref. [4] and the thermoelastic results satisfy the conditions of self-equilibration necessary for the finite-width laminate subjected to free thermal deformation. Integration of the stress σ_x over the laminate cross-section in the y–z plane is shown to converge to zero as the number of Fourier terms is increased. While the exact solution for mechanical loading is known to exhibit singular behavior, non-convergence of the interlaminar shearing strain is also seen to occur at the intersection of the free edge and planes between lamina of $+\theta$ and $-\theta$ orientation under thermal loading. The analytical results show excellent agreement with the finite-element predictions for the same boundary-value problem.

1. Introduction

The investigation of interlaminar phenomena in composite laminates has been underway for many years by numerous investigators including one of the authors of the present study [1–6]. A comprehensive and exhaustive review of this subject by Mittelstedt and Becker is presented in Ref. [6]. Notable references from that work dealing with free edge effects in the angle-ply laminate include [8–11]. Several earlier works also discuss free edge effects due to thermal loading, namely [12–15]. While the development in Ref. [5] is comprehensive in its ability to treat the general symmetric laminate, the more restricted model presented in the following work provides a solution for the symmetric angle-ply laminate that can be directly employed by practitioners who lack the advanced mathematical methods of Mittlestedt and Becker.

The earliest of the investigations of free-edge phenomena in composite laminates focused on the behavior of the finite-width laminate subjected to uniform axial extension [1]. The mechanism of interlaminar load transfer near the free-edge revealed that interlaminar stresses developed within a boundary-layer near the free edge for laminates consisting of an equal number of lamina of $+\theta$ and $-\theta$ orientation with respect to the axial direction and arranged symmetrically about the laminate mid-plane. The interlaminar phenomenon for these laminates, typically referred to as "angle-ply" laminates, involved three primary stress components: the axial stress, σ_x, the in-plane shearing stress, τ_{xy} and the interlaminar shearing stress, τ_{xz}, where x is the laminate axial direction, y is the laminate transverse direction and z is the laminate thickness coordinate, as shown in Fig. 1. Here the components u, v and w describe the displacement components in the x, y and z directions.

Near the free-edge of the angle-ply laminate, a gradient in the in-plane shear stress is developed as a result of the traction free boundary condition requiring the in-plane shear stress to vanish at the edge. Further, the gradient in the in-plane shearing stress within the boundary-layer near the free-edge of each of the lamina also required a gradient in the interlaminar shearing stress in the thickness coordinate, z, to satisfy the equilibrium equation in the axial direction. If the stress components are taken as independent of the axial coordinate, x, then an interlaminar shearing stress is shown to be induced by the gradient in-plane shearing stress. However, it is important to note that the in-plane shearing stress within each of the lamina is the result of the shear-coupling terms with compliance matrix, S_{i6} ($i = 1$–3) of the individual lamina and

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Nomenclature

Symbol	Term
β	coefficient
τ	coefficient (m)
α_1, α_2, α_3	lamina coefficients of thermal expansion (1/°C)
α_x, α_y, α_{xy}	coefficients of thermal expansion (1/°C)
$\bar{\alpha}_x, \bar{\alpha}_y, \bar{\alpha}_z$	laminate coefficients of thermal expansion (1/°C)
σ_x, σ_y, σ_z	normal components of the stress tensor (Pa)
τ_{xy}, τ_{xz}, τ_{yz}	shearing components of stress tensor (Pa)
ε_x, ε_y, ε_z	normal components of the strain tensor
γ_{xy}, γ_{xz}, γ_{yz}	shearing components of the strain tensor
θ	angle between x and 1 axes (°)
C_{ij}	elasticity stiffness matrix (GPa)
S_{ij}	elasticity compliance matrix (GPa^{-1})
h_0	lamina thickness (m)
b	laminate half width (m)
c	cos θ
m	number of lamina
n	number of Fourier terms
s	sin θ
$U(y,z)$	axial displacement function of y and z (m)
$V(y,z)$	transverse displacement function of y and z (m)
$W(y,z)$	normal displacement function of y and z (m)
$U_{,y}$	derivative of U with respect to y
$U_{,z}$	derivative of U with respect to z
ΔT	change in temperature (°C)

that the lamina shear coupling compliance terms are a function of the fiber orientation, θ.

2. Thermoelastic theoretical development

Consider an anisotropic material with a single plane of elastic symmetry. The components of the engineering stress and strain are related through the $S_{ij}(i,j = 1–6)$, terms of the elasticity compliance matrix and the $C_{ij}(i,j = 1–6)$, terms of the elasticity stiffness matrix as shown in Eqs. (1)a–b. The stiffness and compliance matrix terms are also functions of the lamina orthotropic material properties and fiber orientation of each lamina:

$$\begin{bmatrix} \varepsilon_x - \alpha_x \\ \varepsilon_y - \alpha_y \\ \varepsilon_z - \alpha_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} - \alpha_{xy} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ S_{12} & S_{22} & S_{23} & 0 & 0 & S_{26} \\ S_{13} & S_{23} & S_{33} & 0 & 0 & S_{36} \\ 0 & 0 & 0 & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{45} & S_{55} & 0 \\ S_{16} & S_{26} & S_{36} & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} \tag{1a}$$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x - \alpha_x \\ \varepsilon_y - \alpha_y \\ \varepsilon_z - \alpha_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} - \alpha_{xy} \end{bmatrix} \tag{1b}$$

When the stress components are assumed independent of the axial coordinate, x, it has been shown that the displacement–equilibrium equations may be represented in the following form where the displacements U, V and W are functions only of the transverse coordinates y and z [1], where the comma denotes partial differentiation.

$$C_{66}U_{,yy} + C_{55}U_{,zz} + C_{26}V_{,yy} + C_{45}V_{,zz} + (C_{36} + C_{45})W_{,yz} = 0 \tag{2a}$$

$$C_{26}U_{,yy} + C_{45}U_{,zz} + C_{22}V_{,yy} + C_{44}V_{,zz} + (C_{23} + C_{44})W_{,yz} = 0 \tag{2b}$$

$$(C_{45} + C_{36})U_{,yz} + (C_{23} + C_{44})V_{,yz} + C_{33}W_{,zz} = 0 \tag{2c}$$

For the angle-ply laminate, Eqs. (2)a–c reduce to a single, separable partial-differential equation when the laminate is subjected to uniform axial extension [4]. The primary assumptions in this

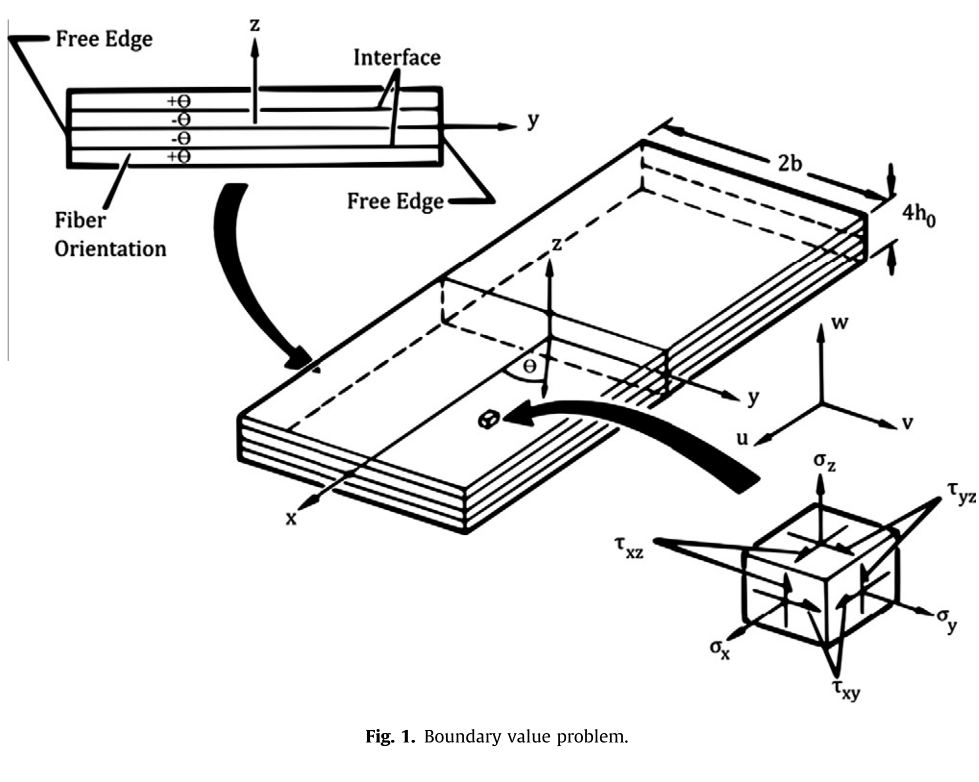

Fig. 1. Boundary value problem.