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An algorithm for determination of the fracture angle for the three-dimensional Puck matrix failure criterion for UD composites

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ABSTRACT

A 3D matrix failure algorithm based upon Puck's failure theory has been developed. The problem of calculating the orientation of a potential fracture plane, which is necessary to assess the onset of matrix failure, has been addressed. Consequently, a fracture angle search algorithm is proposed. The developed algorithm incorporates a numerical search of function extremes which minimises the required computational time for finding the accurate orientation of a potential fracture plane. For illustration, the algorithm together with the three-dimensional Puck failure model has been implemented in LS-DYNA explicit FE code. The fracture angle search algorithm is verified using a virtual uniaxial compression test.

1. Introduction

The development of physically based failure criteria for inter fibre failure (IFF) in long fibre-reinforced polymer composites has been the focus of research for many years. The first promising approach was proposed by Hashin [1] in 1980, who developed failure criteria for plane stress based on the Mohr Coulomb failure theory. The criteria stated that failure was caused by the stresses acting on an inclined fracture plane. However, Hashin did not pursue the calculation of the orientation of that plane because of the required computational effort. Further sound physical basis of the failure theory was posed by Puck [2] who extended the model, which now distinguishes three IFF modes: (a) a tensile and (b) a compressive shear matrix failure, in both of which the crack is perpendicular to direction 2–2, as well as (c) a more complex failure mode in which the fracture plane rotates about 1–1 axis to form a wedge which can cause fibre failure in adjacent layers. Further suggestions for an extension to a 3D state of stress were made in [2]. An extensive experimental study in [3] verified the failure criteria for cases with plane stress states and three-dimensional states of stress.

Initially, Puck's model was not well recognised in the research community. The reason was the large number of unknown parameters and the computationally expensive search of the fracture plane orientation. In order to simplify the application of the failure model, Puck proposed pragmatic solutions for some of the parameters in [4]. As the theory was ranked very highly in the world wide failure exercise (WWFE) [5,6] the model attracted additional attention and further development was undertaken by Davila and Camanho [7], Pinho et al. [8] and Greve and Pickett [9].

The computationally expensive search for the angle of the fracture plane still remains a limiting factor. The application of the model in explicit finite element analysis (FEA) requires a reliable, accurate, yet numerically efficient fracture plane orientation search algorithm. For plane stress, analytical formulations are already available [4]. The fracture angle for three-dimensional states of stress, however, cannot be expressed in a closed form. Therefore, a numerical search procedure needs to be employed. So far, no efficient algorithms for finding the fracture plane angle for three-dimensional states of stress have been proposed in open literature.

This paper introduces a computationally efficient fracture angle search algorithm of a full 3D Puck failure theory for IFF. The algorithm has been verified using a virtual uniaxial compression test and the capabilities of the overall model have been presented.

2. The Puck failure criterion for IFF

The Puck IFF criteria are valid for UD composite laminates. The UD ply is treated as transverse isotropic and is assumed to behave in a brittle manner. The key idea of the Puck failure model is the assumption of a Mohr–Coulomb type of failure for loading transverse to the fibre direction. Failure is assumed to be caused by the normal and shear stresses which are acting on the stress action plane (σ_n, τ_{n1} and τ_{nt}, see Fig. 1). Positive normal stress on this plane promotes fracture while negative normal stress increases

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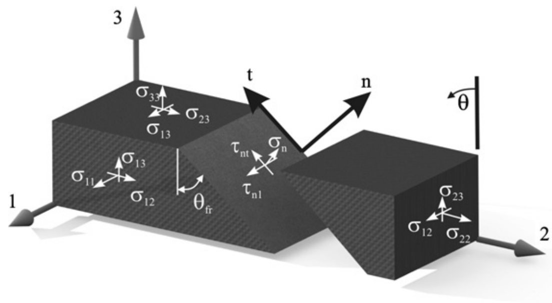

Fig. 1. Definition of the fracture plane by the fracture angle θ_{fr} for the Puck IFF model.

the material's shear strength thus impeding fracture. Pucks stress based failure criteria enable the calculation of the material exposure e as a failure indicator. Failure occurs under the following condition

$$e = 1. \tag{1}$$

The values of e range between 0 (which denotes that material is not loaded) to 1 (which denotes the onset of IFF). A material exposure above 1 is physically inadmissible and denotes the initiation of damage of the material. The material exposure e is a function of the stress state σ and the orientation of the stress action plane against the thickness direction θ.

$$e(\theta, \sigma) \tag{2}$$

Fracture will occur on the stress action plane where $e(\theta, \sigma)$ has a global maximum. This plane is called the fracture plane. The angle of the fracture plane is called the fracture angle θ_{fr}. The definition of fracture plane and fracture angle θ_{fr} is illustrated in Fig. 1.

Puck's criteria define a master failure surface on the fracture plane. Only the stresses which act on that plane (Mohr's fracture plane stresses σ_n, τ_{n1} and τ_{nt}) are assumed to contribute to IFF. The fracture plane stresses are obtained by rotating the three-dimensional stress tensor from material coordinates to the fracture plane. The relationship between the stresses in the ply coordinate system and the Mohr stresses in the inclined fracture plane reads

$$\begin{pmatrix} \sigma_1 \\ \sigma_n \\ \sigma_t \\ \tau_{n1} \\ \tau_{nt} \\ \tau_{1t} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c^2 & s^2 & 2cs & 0 & 0 \\ 0 & s^2 & c^2 & -2cs & 0 & 0 \\ 0 & 0 & 0 & c & 0 & s \\ 0 & -sc & sc & 0 & c^2 - s^2 & 0 \\ 0 & 0 & 0 & -s & 0 & c \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{13} \end{pmatrix} \tag{3}$$

with

$$\begin{aligned} c &= \cos(\theta_{fr}) \\ s &= \sin(\theta_{fr}) \end{aligned} \tag{4}$$

From Fig. 1 it can be seen that only σ_n, τ_{n1} and τ_{nt} contribute to IFF. The fracture plane tractions are given by

$$\begin{aligned} \sigma_n &= \sigma_{22}\cos^2\theta_{fr} + \sigma_{33}\sin^2\theta_{fr} + 2\sigma_{23}\cos\theta_{fr}\sin\theta_{fr}, \\ \tau_{n1} &= \sigma_{12}\cos\theta_{fr} + \sigma_{13}\sin^2\theta_{fr} \\ \tau_{nt} &= -\sigma_{22}\sin\theta_{fr}\cos\theta_{fr} + \sigma_{33}\sin\theta_{fr}\cos\theta_{fr} + \sigma_{23}(\cos^2\theta_{fr} - \sin^2\theta_{fr}). \end{aligned} \tag{5}$$

From Eq. (5) it is clear that all components of stress tensor, except σ_{11}, contribute to IFF.

The master failure surface on the fracture plane is defined in terms of Mohr-Coulomb fracture plane stresses thus yielding the following failure criteria

$$\begin{aligned} e &= \left(\frac{\sigma_n}{R_n}\right)^2 + \left(\frac{\tau_{n1}}{R_{n1} - p_{n1}\sigma_n}\right) + \left(\frac{\tau_{n1}}{R_{nt} - p_{nt}\sigma_n}\right)^2 = 1 \quad \text{for } \sigma_n \geq 0 \\ e &= \left(\frac{\tau_{n1}}{R_{n1} - p_{n1}\sigma_n}\right)^2 + \left(\frac{\tau_{n1}}{R_{nt} - p_{nt}\sigma_n}\right)^2 = 1 \quad \text{for } \sigma_n < 0 \end{aligned} \tag{6}$$

In order to evaluate Eq. (6) it is essential that θ_{fr} is known. The parameters in Eq. (6) are defined as follows (see [4]):

- R_n: resistance of the fracture plane against normal failure
- R_{n1}, R_{nt}: resistance of the fracture plane against shear
- p_{n1}, p_{nt}: slope parameters representing internal friction effects (Mohr–Coulomb type of failure).

Unlike traditional failure criteria, Puck uses fracture plane resistances R. The fracture plane resistance is the resistance of the material against fracture caused by only one component of stress acting on that plane [4]. Two of these values can be obtained directly from simple uniaxial or shear experiments and are identical to the respective strength properties

$$\begin{aligned} R_n &= Y_t, \\ R_{n1} &= S_{12}. \end{aligned} \tag{7}$$

The fracture plane resistance R_{nt} usually cannot be measured directly. The reason is, that unidirectional fibre-reinforced composite materials (e.g., GFRP CFRP) when subjected to a pure transverse shear loading (σ_{23}) fail at an angle $\theta_{fr} = 45°$. In order to assume $R_{nt} = S_{23}$ the failure must happen in the same plane where σ_{23} is acting as a single stress (e.g., $\theta_{fr} = 0°$ or $\theta_{fr} = 90°$). In fact, what could be measured as shear strength S_{23} denotes not a pure shear failure, but a failure due to single normal tensile stress acting on the fracture plane [2] (see stress state1 in Table 2). Puck proposes to calculate R_{nt} from uniaxial compression tests. Specimens loaded uniaxially transverse to the fibre tend to fail by a shear failure on a fracture plane which is inclined by θ^0_{fr}. This and the experimentally observed compressive stress at failure Y_c allow calculating the stress state at failure on the fracture plane. Further assumption of a Mohr–Coulomb type failure allows for the shear stress at failure τ_{nt} to be obtained in the case of $\sigma_n = 0$. This is the missing fracture plane resistance R_{nt} (see Eq. (8)).

The slope parameters p_{n1} and p_{nt} characterise the slope of the fracture envelope at $\sigma_n = 0$ and can be derived experimentally by combined loading experiments [4].

The experimental data necessary to define fully the failure surface is usually not available. Puck gives some pragmatic solutions for the parameters to be derived from simple uniaxial compression experiments [4]. The "missing" parameters are evaluated as follows:

$$\begin{aligned} R_{nt} &= \frac{Y_c}{2\tan\theta^0_{fr}}, \\ p_{nt} &= -\frac{1}{2\tan(2\theta^0_{fr})} \\ p_{n1} &= p_{nt}\frac{R_{n1}}{R_{nt}} \end{aligned} \tag{8}$$

This fully defines the master failure surface. An example of a master failure surface obtained using the data in Table 1 is plotted in Fig. 2. The failure surface is open for negative σ_n because compressive stress σ_n impedes IFF.

Table 1
Typical properties for a carbon epoxy composite [3]

Tensile strength 2 direction	Y_t	59.1 MPa
Compressive strength 2 direction	Y_c	231.2 MPa
In plane shear strength	S_{12}	98.4 MPa
Fracture angle for pure compression	θ^0_{fr}	51°