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Fracture analysis of functionally graded materials by a BEM

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Abstract

In this paper, crack analysis in two-dimensional (2D), continuously nonhomogeneous, isotropic and linear elastic functionally graded materials (FGMs) is presented. For this purpose, a boundary element method (BEM) based on a boundary-domain integral equation formulation is developed. An exponential variation with spatial variables is assumed for Young's modulus of the FGMs, while a constant Poisson's ratio is considered. Fundamental solutions for homogeneous, isotropic and linear elastic solids are applied in the formulation. To avoid displacement gradients in the domain integral, normalized displacements are introduced. By using the radial integration method, the domain integral is transformed into boundary integrals over the global boundary. The normalized displacements in the domain integral are approximated by a combination of radial basis functions and polynomials in terms of global coordinates, which leads to a meshless scheme. Special attention of the analysis is devoted to the computation of the most important crack-tip characterizing parameters of cracked FGMs, namely the stress intensity factors. To show the effects of the material gradation on the stress intensity factors, numerical examples are presented and discussed.

Keywords: A. Functional composites; B. Fracture; B. Mechanical properties; C. Crack; C. Computational mechanics

1. Introduction

In recent years, a new class of composite materials, the so-called functionally graded materials (FGMs), attracted many research interests in materials and engineering sciences [1,2]. FGMs are advantageous over classical homogeneous materials with only one material constituent, because FGMs consist of more material constituents and they combine the desirable properties of each constituent. As a representative example for FGMs, we just mention the metal/ceramic FGMs, which are compositionally graded from a ceramic phase to a metal phase. Metal/ceramic FGMs can incorporate advantageous properties of both ceramics and metals such as the excellent heat, wear, and corrosion resistances of ceramics and the high strength, high toughness, good machinability and bonding capability of metals without severe internal thermal stresses. However, ceramics have a brittle nature, and microcracks or crack-like defects are often induced in the fabrication process or under the in-service loading conditions. Thus, fracture and fatigue analysis of FGMs is an important research issue to the design, optimization, and novel engineering applications of FGMs. For cracked FGMs with general geometry and loading conditions, advanced numerical methods have to be applied, because of the high mathematical complexity of the corresponding governing partial differential equations with variable coefficients, and because the most available analytical methods can be successfully applied to cracked FGMs only with very simple geometry and loading conditions. In this context, we just mention the singular integral equation method [3–7], the classical finite element method (FEM) [8–15], the graded finite element method [16–19], the extended finite element method (XFEM) [20], the element-free Galerkin method (EFG) [21,22], the boundary

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integral equation method (BIEM) or boundary element method (BEM) [23–27], and the meshless Petrov–Galerkin method (MLPG) [28–31].

Although the BEM has been successfully applied to homogeneous, isotropic and linear elastic solids for many years, its application to FGMs is yet very limited due the fact that the corresponding fundamental solutions or Green's functions for general FGMs are either not available or mathematically too complex [32,33]. The nonhomogeneous nature of FGMs prohibits an easy construction and implementation of fundamental solutions for general FGMs.

In this paper, crack analysis in 2D, continuously nonhomogeneous, isotropic and linear elastic FGMs is presented. For this purpose, a boundary-domain integral equation formulation is applied. For simplicity, an exponential variation of Young's modulus and constant Poisson's ratio are assumed. Fundamental solutions for homogeneous, isotropic and linear elastic solids are applied in the present formulation, which results in a boundary-domain integral equation formulation due to the materials nonhomogeneity. To avoid displacement gradients in the domain integral, normalized displacements are introduced. The radial integration method of Gao [34,35] is applied to convert the arising domain integral into boundary integrals over the global boundary of the cracked solids. Basis functions consisting of a combination of radial basis functions and polynomials in terms of global coordinates are used to approximate the normalized displacements in the domain integral. In this manner, a meshless scheme is obtained, which requires only conventional boundary discretization and additional interior nodes instead of cells or meshes. An advantage of the present BEM is that it is easy to implement and can be easily incorporated into an existing BEM code for homogeneous, isotropic and linear elastic solids. Special attention of the analysis is devoted to the investigation of the material gradation on the stress intensity factors. Numerical examples for cracks parallel and perpendicular to the material gradation are presented and discussed.

2. Boundary-domain integral equations

We consider 2D, continuously nonhomogeneous, isotropic and linear elastic FGMs. In the absence of body forces, the equilibrium equations are given by

$$\sigma_{ij,j} = 0, \tag{1}$$

where σ_{ij} represents the stress tensor, a comma after a quantity represents spatial derivatives and repeated indexes denote summation. It is assumed that the Young's modulus $E(x)$ of the FGMs depends on Cartesian coordinates while Poisson's ratio v is constant. In this case, the elasticity tensor $C_{ijkl}(x)$ can be written as

$$C_{ijkl}(\boldsymbol{x}) = \mu(\boldsymbol{x}) C^0_{ijkl}, \tag{2}$$

where

$$\mu(\boldsymbol{x}) = \frac{E(\boldsymbol{x})}{2(1+v)}, \qquad C^0_{ijkl} = \frac{2v}{1-2v}\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}. \tag{3}$$

In Eqs. (2) and (3), $\mu(\boldsymbol{x})$ denotes the shear modulus and δ_{ij} represents the Kronecker delta.

The stress tensor σ_{ij} and the displacement gradients $u_{k,l} = \partial u_k/\partial x_l$ are related by the generalized Hooke's law

$$\sigma_{ij} = C_{ijkl} u_{k,l} = \mu(\boldsymbol{x}) C^0_{ijkl} u_{k,l}. \tag{4}$$

The traction vector t_i on the boundary of the considered domain is related to the stress components by

$$t_i = \sigma_{ij} n_j, \tag{5}$$

where n_j is the outward unit normal vector to the boundary Γ of the domain Ω.

The weak-form of the equilibrium Eq. (1) can be written as

$$\int_\Omega \sigma_{jk,k} \cdot U_{ij} \mathrm{d}\Omega = 0, \tag{6}$$

where $U_{ij}(\boldsymbol{x},\boldsymbol{y})$ is the weight or test function. Substitution of Eq. (4) into Eq. (6) and application of Gauss's divergence theorem yield

$$\int_\Gamma U_{ij} t_j \mathrm{d}\Gamma - \int_\Gamma T_{ij} \mu u_j \mathrm{d}\Gamma + \int_\Omega C^0_{rsjl} U_{ir,sl} \mu u_j \mathrm{d}\Omega + \int_\Omega C^0_{rsjl} U_{ir,s} \mu_{,l} u_j \mathrm{d}\Omega = 0, \tag{7}$$

where

$$T_{ij} = \Sigma_{ijl} n_l, \tag{8}$$

$$\Sigma_{ijl} = C^0_{rsjl} U_{ir,s} = \frac{2v}{1-2v} U_{ik,k}\delta_{jl} + U_{ij,l} + U_{il,j}. \tag{9}$$

For the weight function $U_{ij}(\boldsymbol{x},\boldsymbol{y})$, we choose the displacement fundamental solutions for homogeneous, isotropic and linear elastic solids, which satisfy the following partial differential equations

$$C^0_{rsjl} U_{ir,sl} = -\delta_{ij}\delta(\boldsymbol{x}-\boldsymbol{y}), \tag{10}$$

where $\delta(\boldsymbol{x}-\boldsymbol{y})$ is the Dirac delta function. The solution $U_{ij}(\boldsymbol{x},\boldsymbol{y})$ of Eq. (10) is given by the Kelvin's displacement fundamental solutions for homogeneous, isotropic and linear elastic solids with $\mu = 1$, which can be written as [36]

$$U_{ij} = -\frac{1}{8\pi(1-v)}[(3-4v)\delta_{ij}\ln(r) - r_{,i}r_{,j}], \tag{11}$$

where $r = |\boldsymbol{x}-\boldsymbol{y}|$. Substitution of Eq. (10) into Eq. (7) and application of the sifting property of the Dirac delta function lead to

$$\tilde{u}_i(\boldsymbol{y}) = \int_\Gamma U_{ij}(\boldsymbol{x},\boldsymbol{y}) t_j(x) \mathrm{d}\Gamma - \int_\Gamma T_{ij}(\boldsymbol{x},\boldsymbol{y}) \tilde{u}_j(\boldsymbol{x}) \mathrm{d}\Gamma + \int_\Omega V_{ij}(\boldsymbol{x},\boldsymbol{y}) \tilde{u}_j(\mathbf{x}) \mathrm{d}\Omega, \tag{12}$$