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Optimization of course locations in fiber-placed panels for general fiber angle distributions

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ABSTRACT

Fiber-reinforced composites are usually designed using constant fiber orientation in each ply. In certain cases, however, a varying fiber angle might be favorable for structural performance. This possibility can be fully utilized using tow placement technology. Because of the fiber angle variation, tow-placed courses may overlap and ply thickness will build-up on the surface. This thickness buildup affects manufacturing time, structural response, and surface quality of the finished product.

This paper will present a method for designing composite plies with varying fiber angles with composite plates or panels. The thickness build-up within a ply is predicted as function of ply angle variation using a streamline analogy. It is found that the thickness build-up is not unique and depends on the chosen start locations of fiber courses. Optimal fiber courses are formulated in terms of minimizing the maximum ply thickness, maximizing surface smoothness or combining these objectives with and without periodic boundary conditions.

1. Introduction

In industry fiber-reinforced composites are usually designed using a constant fiber orientation in each ply. The fiber angles in these laminates are typically 0°, 90°, and ±45°. Traditionally the choice of these lay-ups was motivated by manufacturability, while nowadays lay-ups with changing or even non-conventional fiber angles are avoided because of the lack of allowables. However, research on composites with a varying in-plane fiber orientation has shown that variable stiffness can be beneficial for structural performance [1–17], because variable-stiffness laminates are able to redistribute the loading more efficiently than constant-stiffness laminates. In most cases curvilinear fiber paths manufactured by tow placement are used to construct the variable-stiffness laminates [4,5,9–11,15,18–20]. Jegley, Tatting and Gürdal [9–11] designed variable-stiffness flat plates with holes and demonstrated their effectiveness by building and testing several specimens.

Due to fiber angle variation, a tow-placed shell typically exhibits gaps and/or overlaps between adjacent courses and ply thickness will change along the surface [9–11,18]. The amount of gap/overlap affects structural response, manufacturing time, and surface quality of the finished product.

This paper presents a method for designing composite plies which have spatially varying fiber angles. Since fiber-reinforced laminates usually consist of multiple plies, optimizations for specific loading conditions result in multiple plies with different fiber angle distributions. The fiber angle distribution per ply can be used as a direct input for the optimization, as is done by most researchers so far [1–20], or it can be obtained in a post processing step, where an optimum laminate stiffness distribution is approximated as closely as possible, as described by Setoodeh et al. [21]. In these optimizations the ply thickness is usually assumed to be constant, without taking into consideration manufacturing issues. In the current paper the fiber angle distribution per ply is assumed given, being one of the plies within an optimized laminate. The thickness build-up is predicted as function of ply angle variation using a streamline analogy. It is found that the thickness build-up is not unique and depends on the chosen start locations of fiber courses. Optimal distributions of fiber courses are formulated in terms of minimizing the maximum ply thickness or maximizing surface smoothness, either with or without periodic boundary conditions. Subsequently the discrete thickness build-up resulting from the tow-placement process can be determined based on the streamline distribution. Results will be compared to the smeared thickness approximation. An overview of the analysis sequence is given in Fig. 1. Finally, a number of applications for the developed methods and suggestions for future research are given.

2. Streamline analogy

For the construction of discrete fiber paths a streamline analogy is being used. For this application each streamline represents the centerline of a course, or if the course width is made infinitely

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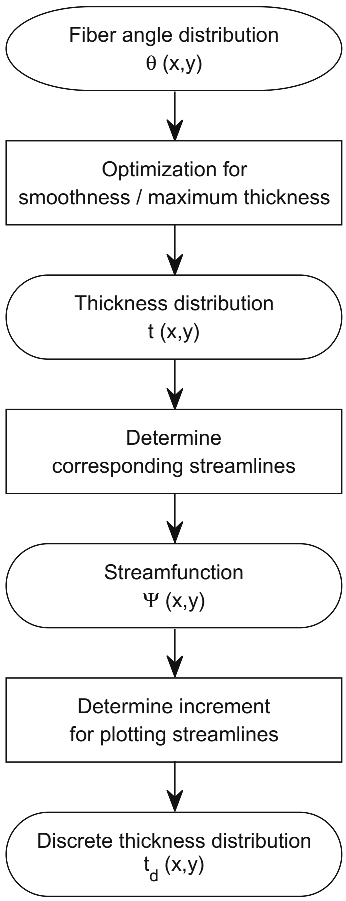

Fig. 1. Analysis sequence.

small each streamline will represent a single fiber. Mathematically a streamline is represented by a stream function

$$\Psi(x, y) = C \tag{1}$$

which connects all the points with a constant value C. For a given fiber angle distribution $\theta(x, y)$, the streamlines can be found by solving the following partial differential equation:

$$\frac{d\Psi}{ds} = \frac{\partial\Psi}{\partial x}\frac{dx}{ds} + \frac{\partial\Psi}{\partial y}\frac{dy}{ds} = \Psi_{,x}\cos\theta + \Psi_{,y}\sin\theta = 0 \tag{2}$$

A unique solution for the stream function (and thus the location of the stream lines) depends on the boundary conditions. Before seeking a solution to the stream function, additional considerations relevant to the physical representation of the fiber paths are in order.

As stated earlier, the streamlines represent the central path of a finite width course. Unless the streamlines are parallel, the successive courses will always overlap each other when no gaps are allowed between them (or alternatively, the gaps will form between the passes if two successive finite width passes are not allowed to overlap). The amount of overlap depends on the distance between the course centerlines. If the distance is decreased, then the overlap area is increased. Although in reality these overlaps are discrete, a first approximation to the amount of overlap could be made by smearing out this discrete overlap to form a continuous thickness distribution. In this case, the smeared thickness, t, will be inversely proportional to the distance between adjacent courses, which can be explained as follows. If a number of N courses with a given width, w, and thickness has a fixed volume V, and if these successive courses are placed closer than the width of the courses, then the total width covered is less then $N \cdot w$, and the thickness has to be increased in order to maintain the same material volume V.

When the distance between two streamlines is $|dn|$, then $t \propto 1/|dn|$ (as explained above). Since $\Psi_{,n} = d\Psi/dn$ and $d\Psi$ between two streamlines is constant according to Eq. (1) the thickness t will be proportional to $\Psi_{,n}$ as follows:

$$t \propto \frac{1}{|dn|} = \frac{1}{d\Psi/\Psi_{,n}} = \frac{\Psi_{,n}}{d\Psi} \propto \Psi_{,n} \tag{3}$$

If $d\Psi$ is assumed to be a unity, then $t = \Psi_{,n}$, which can be used to derive a direct correlation between the thickness distribution and the fiber angle variation (see Appendix A):

$$-\bar{s}\overline{\nabla}(\ln t) = \bar{n}\overline{\nabla}\theta \tag{4}$$

in which $\bar{s}$ and $\bar{n}$ represent the tangent and normal vectors to a streamline, respectively, as shown in Fig. 2. The physical explanation of Eq. (4) is that the change in thickness along a streamline depends on the change of the fiber orientation perpendicular to that streamline. Since both vectors $\bar{s}$ and $\bar{n}$ depend on the given fiber angle distribution $\theta(x, y)$, the only unknown in Eq. (4) is the thickness. Hence, the thickness can now be determined by solving this equation, but since it is a differential equation boundary conditions are needed in order to obtain a unique solution. In accordance with streamline theory, boundary conditions are only needed at the inflow boundary, where the inflow boundary is arbitrarily defined by:

$$\bar{s} \cdot \overline{N} \leqslant 0 \tag{5}$$

where $\bar{s}$ is the vector tangent to the streamline and $\overline{N}$ is the outward normal vector to the boundary, as shown in Fig. 2. By changing the thickness at the inflow boundary, the thickness distribution inside the domain and at the outflow boundaries will change.

3. Determining boundary conditions

There exist an infinite number of possible boundary conditions for which the thickness distribution associated with the streamlines can be found, but the most difficult part is to find the ones that are physically sensible for the problem in hand. In this paper the boundary conditions are established such that they fulfill a certain optimality condition. The optimality conditions to be demonstrated in this paper are minimization of maximum thickness, maximization of smoothness, and a combination of these two. In addition, constraints such as periodicity of the boundary conditions can be enforced as well.

3.1. General solution

By using the following change of variables: $\tau = \ln t$, Eq. (4) becomes:

$$-\bar{s}\overline{\nabla}\tau = \bar{n}\overline{\nabla}\theta \tag{6}$$

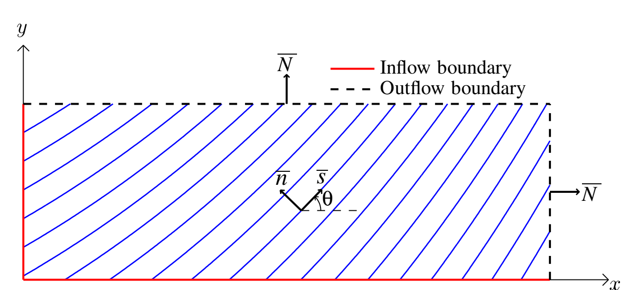

Fig. 2. Streamline definitions.