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Stochastic approach to multiple cracking in composite systems based on the extreme-values theory

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ABSTRACT

A stochastic model of fragmentation of brittle constituents, which has been pioneered for ceramic matrix composites, is revisited. It consists in a stochastic description of the brittle fracture of successive fragments having a decreasing size. Fracture of a fragment is caused by its most severe flaw. The approach is based on the low extremes of fragment flaw strength distributions.

The paper is aimed at assessing the approach and its potential application to various composite systems containing brittle constituents, such as polymer matrix composites or multi-layers. Expressions for fragments and fibers failures involve various random variables. The influence of the random variables on tensile stress–strain behaviour predictions was investigated. Fragment strength–size relationships were established using tensile tests performed on C/SiC minicomposites in the chamber of a scanning electron microscopy (SEM). Experimental data and predictions validated the approach. Important implications for the prediction of multiple cracking and resulting stress–strain behaviour are discussed. Then simple analytical expressions were derived.

1. Introduction

The future for structural materials is in hybrid systems made of small size elements. Introducing small volumes of materials implies reducing the size of processing flaws, which makes the mechanical strength of constituents to increase tremendously, as well as the performance of the system. Current examples of such hybrid systems include fiber reinforced composites (FRC) and multi-layers. In FRC, the fractions of fibers and matrix are comparable (around 50% each), fibers have small diameters (7–16 μm in ceramic or carbon matrix composites), whereas the matrix is also a few microns thick. Constituents display contrasting properties, such as stiff fibers against compliant matrices (polymer matrix reinforced with ceramic or carbon fibers: PMCs) or strong fibers against weak matrices (ceramics reinforced with ceramic or carbon fibers: CMCs). In PMCs, fibers fail first whereas the opposite is observed in CMCs. Both systems are sensitive to multiple cracking when the applied load is increased (Figs. 1 and 2). The loads are transferred through the surviving constituent. This phenomenon may be observed in multi-layers also.

Thus, modelling properly multiple cracking is of primary importance to calculate the associated non-linear deformations, with a view to component or material design. There have been several attempts in the literature aimed at simulating fiber or matrix fragmentation. The approaches were based either on a unique distribution of strengths for the entire volume of constituent which experiences cracking [1–9], or on flaw density functions pertaining to fragments [10–16].

In the former, the Weibull equation [1–6] or alternative forms [7–8] were employed. This implies that the low extreme of the flaw density function is considered only, although the biggest flaws are not the only fracture inducing ones as fragmentation proceeds under increasing load. Experimental results obtained during fiber fragmentation tests showed that this approach may provide satisfactory predictions only for low stresses, far from the saturation of fragmentation [7–9,12]. This result is logically expected since the low strength extreme is considered. This distribution may be accepted for the first cracks from the most severe flaws.

Despite these limitations, many researchers rely on this approach to construct a Monte Carlo simulation of fragmentation. They proceed as follows:

- the volume V of fragmenting constituent (fiber or fiber coating (matrix)) is divided into a large number of identical elements with volume V_e,
- the strength of volume V is assumed to follow the Weibull model.

$$P = 1 - \exp\left[-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right] \tag{1}$$

where m and σ_0 are the shape and scale parameters. As mentioned above, this equation refers to the low extreme of the

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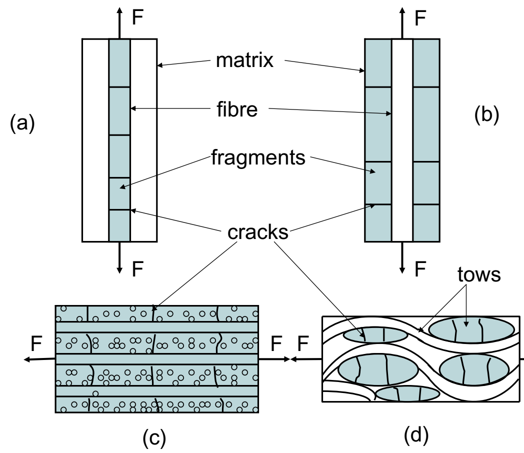

Fig. 1. Schematic diagrams showing examples of multiple cracking in various composite systems: (a) fiber fragmentation in a polymer matrix composite, (b) matrix fragmentation in a ceramic matrix composite, (c) in a cross ply laminate, (d) fragmentation of transverse tows in a woven composite.

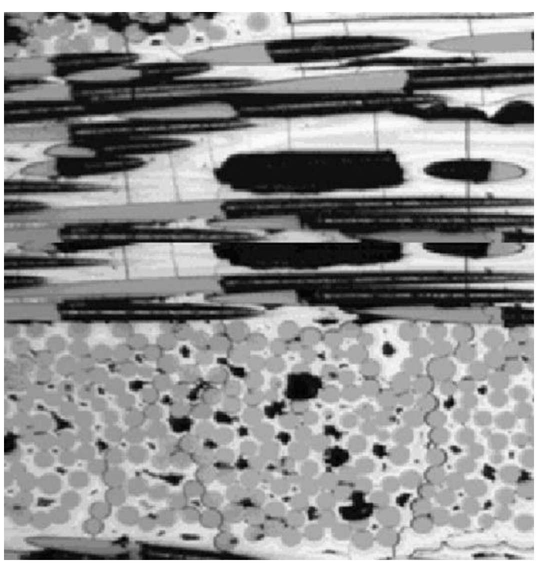

Fig. 2. SEM micrograph showing multiple cracks in the matrix of longitudinal tows, and of transverse tows of a woven ceramic matrix composite. Note that the cracks propagated through the tows.

flaw strength distribution. It does not represent the entire population of fragment inducing flaws.

- the strength of each element of volume V_e is derived from (1), by generating a random value which is substituted for *P*,
- given the applied net stress on the fiber, broken elements are searched for.

Thus, it appears that the series of cracking stresses is given by Eq. (1), applied to identical volume elements V_e It corresponds to a reduced population of fracture inducing flaws.

In the second approach, the phenomenon of fragment generation is addressed. It is considered that each fragment results from the failure of a parent fragment. The fracture inducing flaw corresponds to the low extreme of the flaw density function pertaining to the parent fragment [13–16]. This approach allowed sound predictions of the tensile stress–strain behaviour of SiC/SiC or C/SiC minicomposites (i.e. SiC matrix composites reinforced by single

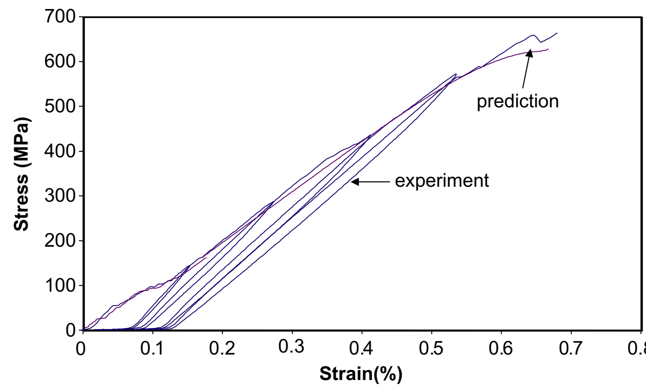

Fig. 3. Example of stress–strain curves predicted using the model for P_{Mi} = 0.5; and obtained experimentally (C/SiC minicomposite) [16].

tows of SiC or C fibers) [14,16,17]. Fig. 3 shows an example of agreement of prediction with experimental behaviour. It is worth pointing out that this is not model identification but, instead, validation, since the characteristics of constituents were determined independently. But, the approach is based on the above assumptions plus a simplifying one:

- statistical distribution of fragment strengths is still pertinent as they become smaller and smaller until saturation, which implies that they contain a sufficient amount of flaws,
- failure of fragments is caused by the weakest flaws,
- average strengths of fragments were used, although strength is a random variable.

The paper discusses this latter approach of multiple cracking. It was validated using (i) predictions of stress–strain behaviour for microcomposites (i.e. composites reinforced by single fibers) for various sets of random variables including the most general case, and (ii) experimental strength–fragment size relations derived from tensile tests performed under SEM microscopy on C/SiC minicomposites. Then, simple analytical expressions were derived.

2. Model

2.1. General equations of multiple cracking

Brittle failure of the weakest constituent is described by the following failure probability equation [18–20]:

$$P = 1 - \exp - \left[\int_V dV \int_0^S g(S) dS \right] \tag{2}$$

where g(*S*) is the flaw density function and *S* is the elemental flaw strength.

It is demonstrated that Eq. (1) reduces to the following equation, when the low strength extreme of *g*(*S*) is considered only, and when it is described using a power law [18–20] with constants m and λ_0:

$$P = 1 - \exp \left[- \frac{V}{V_0} K \left(\frac{\sigma_{ref}}{\lambda_0} \right)^m \right] \tag{3}$$

where λ_0 is a scale factor, *m* is a shape parameter. *K* is obtained by integrating the stress-state over the volume V. *K* depends on the probabilistic model which is considered [18–20]. σ_{ref} is a reference stress (peak stress) in *V*. V_0 is the reference volume (V_0 = 1 m^3 when International Units are used).

When the weakest constituent experiences multiple cracking, Eq. (2) applies to brittle failure of fragments. The flaw density