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Maximum curvatures of 0/90 plates under thermal stress: Modelling and experimental validation

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Abstract

In this paper, special attention is paid to the maximum curvatures induced by *uniform* thermal fields in $0_m/90_n$ laminated square plates. The effects of the relative thickness of the 90° ply e_{90} and of the elastic and thermoelastic anisotropy on the curvature tensor are then studied. The conditions for which the curvatures are maximum are established by adopting the polar method within the context of the classical lamination theory. It is found that the location of the maximum curvatures is influenced by the level of *elastic* anisotropy, defined by the polar parameter R_1, while the magnitude of the maximum curvatures is driven by the *thermoelastic* properties, that is, the coefficients of thermal expansion. The capabilities of a geometrical nonlinear model are explored and experimental evidence of maximum curvatures of glass/epoxy composite materials are provided.

Keywords: A. polymer–matrix composites; C. laminate theory; C. Residual stress

1. Introduction

$0_m/90_n$ unsymmetric composite plates under thermal stress deform because of the thermoelastic coupling. Indeed, the curing temperature of such plates is different from the service ambient temperature, therefore the temperature differential ($T_{\mathrm{cure}} - T_{\mathrm{room}}$) produces consistent thermal loads (thermal forces and moments) and curvatures. Curvature measurements are utilized to determine curing stresses ([1,2]). The basic idea is to identify the laminate stacking sequences allowing to perform optimal experiments due to their high sensitivity to external loads. Also, it would be desirable in preliminary design of structures to design plates that have a desired deformed shape when subjected to thermal solicitations.

Vannucci [3,4] studied in detail the phenomenon of elastic and thermoelastic coupling, that is, the structure of **B** and **V** tensors of the classical laminated plate theory (CLPT, [5]). He employed the polar method, making use of invariant quantities of such tensors; in particular, by this analytical method he was able to determine explicit expressions for their maximum norms.

In the present paper, the approach already employed by Vannucci is extended to the curvature tensors k, whose components are maximised for plates belonging to the $0_m/90_n$ family. A geometrical nonlinear model of plates is then used to verify the applicability range of the simple linear theory. The paper is divided as follows: first, the employed models are briefly recalled, then the results of some simulations are presented and, finally, experimental results validating the models are provided.

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2. Model description

2.1. Recall of the polar method and conditions of maximum coupling

The Cartesian components of the anisotropic stiffness tensor, **Q**, of a composite ply may be expressed in their polar form [3] by employing the six parameters T_0, T_1, R_0, R_1, Φ_0 and Φ_1 as follows:

$$\begin{aligned} Q_{xx} &= T_0 + 2T_1 + R_0\cos 4\Phi_0 + 4R_1\cos 2\Phi_1 \\ Q_{xy} &= -T_0 + 2T_1 - R_0\cos 4\Phi_0 \\ Q_{yy} &= T_0 + 2T_1 + R_0\cos 4\Phi_0 - 4\Phi_1\cos 2\Phi_1 \\ Q_{ss} &= T_0 - R_0\cos 4\Phi_0 \\ Q_{xs} &= R_0\sin 4\Phi_0 + 2R_1\sin 2\Phi_1 \\ Q_{ys} &= -R_0\sin 4\Phi_0 + 2R_1\sin 2\Phi_1 \end{aligned} \tag{1}$$

with,

$$\begin{aligned} 8T_0 &= Q_{xx} - 2Q_{xy} + 4Q_{xs} + Q_{yy} \\ 8T_1 &= Q_{xx} + 2Q_{xy} + Q_{yy} \\ 8R_0 e^{4i\Phi_0} &= Q_{xx} - 2Q_{xy} - 4Q_{ss} + Q_{yy} + 4i(Q_{xs} - Q_{ss}) \\ 8R_1 e^{2i\Phi_1} &= Q_{xx} - Q_{yy} + 2i(Q_{xs} + Q_{ss}) \end{aligned} \tag{2}$$

The following statements can be formulated:

– T_0, T_1, R_0, R_i and $\Phi_0 - \Phi_1$ are invariant under a rotation θ with respect to the reference frame,
– a material is isotropic if $R_0 = R_1 = 0$. T_0 and T_1 constitute the isotropic parts of **Q**, while R_0, R_1, Φ_0, Φ_1 constitute its anisotropic part,
– **Q** is orthotropic if and only if $(\Phi_0 - \Phi_1) = k\pi/4$ with k integer.

The equations of the classical plate lamination theory state that [5]:

$$\begin{aligned} \mathbf{N} &= \mathbf{A}\varepsilon_0 + \mathbf{B}k - \mathbf{U}\Delta T \\ \mathbf{M} &= \mathbf{B}\varepsilon_0 + \mathbf{D}k - \mathbf{V}\Delta T \end{aligned} \tag{3}$$

Invariant quantities analogous to those defined in Eq. (2) can be introduced tensors for **A**, **B**, **D**, **U**, **V**, respectively $\bar{T}_0$, $\bar{T}_1$, $\bar{R}_0$, $\bar{R}_1$, $\bar{\Phi}_0$, $\bar{\Phi}_1$ for **A**, $\widehat{T}_0$, $\widehat{T}_1$, $\hat{R}_0$, $\hat{R}_1$, $\hat{\Phi}_0$, $\hat{\Phi}_1$ for **B**, $\tilde{T}_0$, $\tilde{T}_1$, $\tilde{R}_0$, $\tilde{R}_1$, $\tilde{\Phi}_0$, $\tilde{\Phi}_1$ for **D**, $\bar{T}, \bar{R}, \bar{\Phi}$ for **U** and $\widehat{T}, \hat{R}, \hat{\Phi}$ for **V**. The reader can refer to the work by Vannucci et al. [3] for more detail about the exact definition of such quantities. Here, we simply note that tensors **B** and **V** represent, respectively elastic and thermoelastic coupling.

Finally the norm of a fourth order tensor (**A** for instance) can be defined by the following expression:

$$A = \sqrt{\bar{T}_0^2 + 2\bar{T}_1^2 + \bar{R}_0^2 + 4\bar{R}_1^2} \tag{4}$$

In the following sections we will consider laminates with identical plies, for which:

$$\widehat{T}_0 = \widehat{T}_1 = \widehat{T} = 0 \tag{5}$$

According to Vannucci [4], a laminate is *decoupled* if the tensor norms B and V (pertaining, respectively, to tensors **B** and **V**) are equal to zero: it is important to note that – for laminates with identical plies – the condition $\mathbf{B} = 0$ implies $\mathbf{V} = 0$, while the inverse is not true.

Vannucci [4], by using the polar method, established explicitly the conditions of maximum coupling, that can be read as follows:

– the thermoelastic coupling is maximum (max. V) when all plies belonging to each half of the stacking sequence, with respect to the middle plane, have the same orientation differing by an angle equal to $\pi/2$: this result is not influenced by the material properties,
– the conditions of maximum elastic coupling (max. B) are the same as for V only if $\rho = R_0/R_1 < 1$; on the contrary, max. B is reached with plates whose halves differ by an angled δ equal to:

$$\delta = \frac{1}{2}\arccos\left(-\frac{1}{\rho^2}\right) \tag{6}$$

This time, material properties have an important influence on the maximum coupling. In fact, for laminates with $\rho > 1$, B and V are not maximised by the same stacking sequence.

2.2. Maximum curvatures of $0_m/90_n$ plates

By re-writing the constitutive Eq. (3) using the polar formalism and by employing invariant quantities for the thermal forces and moments we obtain, for a laminate belonging to the $0_m/90_n$ family:

$$\begin{pmatrix} T_N \\ T_N \\ T_M \\ R_M \end{pmatrix} = \begin{pmatrix} 4\bar{T}_1 & 4\bar{R}_1 & 0 & 4\hat{R}_1 \\ 4\bar{R}_1 & 2(\bar{T}_0 + \bar{R}_0) & 4\hat{R}_1 & 0 \\ 0 & 4\hat{R}_1 & 4\tilde{T}_1 & 4\tilde{R}_1 \\ 4\hat{R}_1 & 0 & 4\tilde{R}_1 & 2(\tilde{T}_0 + \tilde{R}_0) \end{pmatrix} \begin{pmatrix} T_\varepsilon \\ R_\varepsilon \\ T_k \\ R_k \end{pmatrix} \tag{7}$$

T_ε, R_ε, T_k, R_k can be determined by inverting Eq. (7) and curvatures k_{xx} and k_{yy} can be obtained by the following equations:

$$T_k = \frac{k_{xx} + k_{yy}}{2}, \quad R_k = \frac{k_{xx} - k_{yy}}{2} \tag{8}$$

It has to be noted that, for laminates belonging to the $0_m/90_n$ family under thermal solicitations the twisting curvature k_{xy} is equal to zero, because there are no shear and twisting thermal solicitations, respectively, N_{xy}^{th} and M_{yx}^{th} to activate them.

The curvatures can be studied as functions of the adimensional thickness (with respect to the total thickness of the plate, e) of the 90° plies, e_{90}, and the value of e_{90} for which curvatures are maximum can be explicitly assessed by means of analytical expressions.

For instance, the maximum k_{xx} is given by a value of e_{90} which is solution of the following equation: