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An investigation of dynamic interaction between multiple cracks and inclusions by TDBEM

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ABSTRACT

In this paper, the dynamic interaction between multiple inclusions and cracks is studied by the time-domain boundary element method (TDBEM). To deal with this problem, two kinds of time-domain boundary integral equations together with the sub-region technique are applied. The cracked solid is divided into homogeneous and isotropic sub-regions bounded by the interfaces between the inclusions and the matrix. The non-hypersingular traction boundary integral equations are applied on the crack-surfaces; while the traditional displacement boundary integral equations are used on the interfaces and the exterior boundaries. In the numerical solution procedure, square-root shape functions are adopted for the crack-opening-displacements to describe the proper asymptotic behavior in the vicinity of the crack-tips. Numerical results for dynamic stress intensity factors are presented for various cases. The effects of the inclusion position, material combinations and multiple micro-cracks on the dynamic stress intensity factors are discussed.

1. Introduction

To improve material performances, a widely used and practical way is introducing strengthening material phases into a matrix to form a multi-phase inhomogeneous material, such as fiber-reinforced materials or particle-reinforced materials. Understanding the fracture behavior of this kind of composite materials is vitally important to develop high performance composites for technological applications. The interaction between pre-existing flaws or cracks and inclusions will bring a significant effect on material properties which led to a great deal of analytical, numerical and experimental explorations in this area. A detailed review about the previous works can be referred to Kitey et al. [1].

Early attempts using analytical approaches were made by Tamate [2], Atkinson [3], Sendeckyj [4] and Erdogan et al. [5,6] for various cases of a crack inside, outside, penetrating or lying on the interface. Following these works, more complicated cases such as concerning more cracks or elliptical inclusions were treated analytically [7–12]. Due to the complexity of the interaction between cracks and inclusions, the analytical solutions were obtained only for limited cases. More general situations were performed by numerical approaches such as finite element method (FEM) [13–17] and boundary element method (BEM) [18–20]. Most existing research works are devoted to static loading conditions. Recently, the dynamic interaction between an inclusion and a nearby moving crack was investigated by a BEM ([22]). In that paper, special attention was paid to the crack trajectory under dynamic loading. But till now, to our knowledge, no detailed investigations on the dynamic interaction between multiple cracks and inclusions have been done.

The aim of this paper is to further extend the time-domain BEM, which has been successfully developed for the dynamic interaction between a crack and an interface by Lei et al. [21], to more general cases including a cluster of cracks and inclusions. Combining the traditional time-domain displacement integral equations (BIEs) for the external boundaries and the interfaces, and the non-hypersingular traction BIEs ([23]) for crack-surfaces in conjunction with the sub-region technique, the more complicated case of a bounded domain with multiple cracks and inclusions under dynamic loading can be well treated. Numerical results for the dynamic stress intensity factors are presented to demonstrate the dynamic interaction effects on the crack-tip field for various locations and material combinations.

2. Problem statement and time-domain BIEs

Consider a two-dimensional (2D) elastic solid containing p cracks and q inclusions as shown in Fig. 1. The deformation of the cracked solid is either in a state of plane strain or plane stress. All constituent materials are assumed to be homogeneous, isotropic and linearly elastic. The matrix is surrounded by the external

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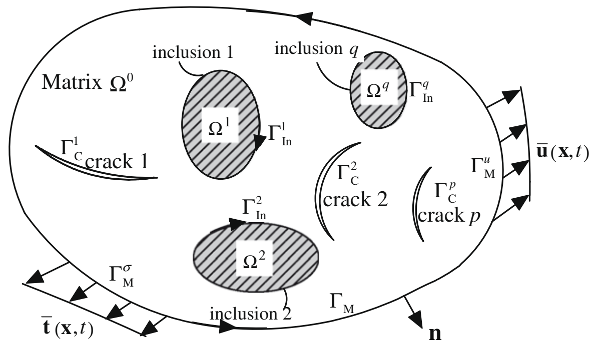

Fig. 1. A solid with a cluster of cracks and inclusions under dynamic loading.

boundary $\Gamma_{\rm M} = \Gamma^\sigma_{\rm M} + \Gamma^u_{\rm M}$, the boundary of the inclusions $\Gamma^i_{\rm In}$ ($i = 1,2,\ldots,q$) and the crack-faces $\Gamma^i_{\rm C}$ ($i = 1,2,\ldots,p$). Here $\Gamma^\sigma_{\rm M}$ and $\Gamma^u_{\rm M}$ denote the parts with prescribed traction $\bar{t}$ and prescribed displacement $\bar{u}$, respectively.

The material parameters are given as follows: shear modulus μ^i, Lamé constants λ^i, Poisson's ratio v^i, mass density ρ^i, shear wave velocity C^i_T and longitudinal wave velocity C^i_L, respectively. The superscript $i = 0$ represents the matrix and $i = 1,2,\ldots,q$ represents ith inclusion.

In the absence of body forces, the cracked solid satisfies the equations of motion

$$\sigma^i_{\alpha\beta,\beta} = \rho^i \ddot{u}^i_\alpha \tag{1}$$

and the Hooke's law

$$\sigma^i_{\alpha\beta} = C^i_{\alpha\beta\gamma\delta} u^i_{\gamma,\delta}. \tag{2}$$

The initial conditions can be written as

$$u^i_\alpha(\mathbf{x},0) = \dot{u}^i_\alpha(\mathbf{x},0) = 0 \tag{3}$$

and the boundary conditions are

$$t_\alpha(\mathbf{x},t) = \bar{t}_\alpha(\mathbf{x},t), \quad \mathbf{x} \in \Gamma^t_{\rm M}, \tag{4}$$

$$u_\alpha(\mathbf{x},t) = \bar{u}_\alpha(\mathbf{x},t), \quad \mathbf{x} \in \Gamma^u_{\rm M}, \tag{5}$$

for external boundary and

$$t_\alpha(\mathbf{x},t) = \sigma_{\alpha\beta}(\mathbf{x},t) n_\beta = \bar{t}^i_\alpha(\mathbf{x},t), \quad \mathbf{x} \in \Gamma^i_{\rm C}, \quad (i = 1,2,\ldots,p), \tag{6}$$

for the crack-faces on which a general load $\bar{t}^i_\alpha(\mathbf{x},t)$ is applied.

The continuity conditions on the interfaces are

$$t^0_\alpha(\mathbf{x},t) = -t^i_\alpha(\mathbf{x},t), \quad u^0_\alpha(\mathbf{x},t) = u^i_\alpha(\mathbf{x},t), \quad \mathbf{x} \in \Gamma^i_{\rm In}, \quad (i = 1,2,\ldots,q), \tag{7}$$

where $\sigma^i_{\alpha\beta}$, u^i_α and t^i_α denote the stress, the displacement and the traction components, $C^i_{\alpha\beta\gamma\delta}$ is the fourth order elasticity tensor, a comma after a quantity designates spatial derivatives, while the superscript dots stand for temporal derivatives. Unless otherwise stated, the conventional summation rule over double indices is applied with Greek indices $\alpha,\beta,\gamma,\delta = 1,2$ for the present 2D problem.

The non-hypersingular time-domain traction BEM presented by Zhang and Gross [23] and the traditional time-domain displacement BEM in conjunction with the sub-region technique are adopted to treat this problem. To apply the time-domain BIEs for a homogeneous, isotropic and linearly elastic domain, the multi-phase system is split from the inclusion interfaces into $q + 1$ separated homogeneous sub-domains including one main sub-domain Ω^0 occupied by the matrix with p cracks and q holes and the remaining q sub-domains Ω^i ($i = 1,\ldots,q$) occupied by the individual inclusions. With $\Gamma^0 = \Gamma_{\rm M} + \sum_{i=1}^q \Gamma^i_{\rm In}$ we have $\partial\Omega^0 = \Gamma^0 + \sum_{i=1}^p \Gamma^i_{\rm C}$.

For the domain Ω^0 containing cracks, the hybrid BEM combining the displacement BEM with the traction BEM is a feasible method to deal with this problem. The following non-hypersingular time-domain traction BIEs:

$$\begin{aligned} t^{\Gamma^i_C}_\alpha(\mathbf{x},t) = {} & C^0_{\alpha\gamma\upsilon\delta} n_\gamma(\mathbf{x}) \int_{\Gamma^0} \left\{ \mathbf{e}_{\delta\varepsilon} \sigma^{G0}_{\beta\varepsilon\upsilon}(\mathbf{x},\mathbf{y},t) * \frac{\partial u^0_\beta}{\partial s}(\mathbf{y},t) + \rho^0 u^{G0}_{\beta\upsilon}(\mathbf{x},\mathbf{y},t) * \ddot{u}^0_\beta(\mathbf{y},t) n_\delta(\mathbf{y}) \right\} ds(\mathbf{y}) \\ & - n_\gamma(\mathbf{x}) \int_{\Gamma^0} \sigma^{G0}_{\alpha\gamma\beta}(\mathbf{x},\mathbf{y},t) * t^0_\beta(\mathbf{y},t) ds(\mathbf{y}) \\ & - C^0_{\alpha\gamma\nu\delta} n_\gamma(\mathbf{x}) \sum_{i=1}^p \int_{\Gamma^i_C} \left\{ \mathbf{e}_{\varepsilon\delta} \sigma^{G0}_{\beta\varepsilon\upsilon}(\mathbf{x},\mathbf{y},t) * \frac{\partial \Delta u^i_\beta}{\partial s}(\mathbf{y},t) + \rho^0 u^{G0}_{\beta\upsilon}(\mathbf{x},\mathbf{y},t) * \Delta\ddot{u}^i_\beta(\mathbf{y},t) n_\delta(\mathbf{y}) \right\} ds(\mathbf{y}) \end{aligned} \tag{8}$$

are separately applied to each crack-surface Γ^i_C, and the traditional displacement BIEs

$$\begin{aligned} c_{\alpha\beta}(\mathbf{x}) u^0_\beta(\mathbf{x},t) = {} & \int_{\Gamma^0} \{ u^{G0}_{\beta\alpha}(\mathbf{x},\mathbf{y},t) * t^0_\beta(\mathbf{y},t) - n_\gamma(\mathbf{y}) \sigma^{G0}_{\beta\gamma\alpha}(\mathbf{x},\mathbf{y},t) * u^0_\beta(\mathbf{y},t) \} ds(\mathbf{y}) \\ & + \sum_{i=1}^p \int_{\Gamma^i_C} n^2_\gamma(\mathbf{y}) \sigma^{G0}_{\beta\gamma\alpha}(\mathbf{x},\mathbf{y},t) * \Delta u^i_\beta(\mathbf{y},t) ds(\mathbf{y}) \end{aligned} \tag{9}$$

are applied to Γ^0.

For the remaining q sub-domains or inclusions, each domain or inclusion is a single-phase material. So the traditional displacement BIEs

$$\begin{aligned} c_{\alpha\beta}(\mathbf{x}) u^i_\beta(\mathbf{x},t) = \int_{\Gamma^i_{\rm In}} \{ & u^{Gi}_{\beta\alpha}(\mathbf{x},\mathbf{y},t) * t^i_\beta(\mathbf{y},t) - n_\gamma(\mathbf{y}) \sigma^{Gi}_{\beta\gamma\alpha}(\mathbf{x},\mathbf{y},t) \\ & * u^i_\beta(\mathbf{y},t) \} {\rm d}s(\mathbf{y}) \end{aligned} \tag{10}$$

can be separately applied to the boundary $\Gamma^i_{\rm In}$ of each sub-domain.

In Eqs. (8)–(10), $\mathbf{x} = (x_1,x_2)$ and $\mathbf{y} = (y_1,y_2)$ represent the source and the observation points, $\sigma^{Gi}_{\beta\gamma\alpha}$ and $u^{Gi}_{\beta\alpha}$ are the 2D elastodynamic fundamental solutions for stresses and displacements, and an asterisk $*$ denotes Riemann convolution which is defined by

$$g(x,t) * h(x,t) = \int_0^t g(x,t-\tau) h(x,\tau) d\tau. \tag{11}$$

The term $c^i_{\alpha\beta}(\mathbf{x})$ is a constant matrix which depends on the smoothness of the boundary and reduces to $0.5\delta_{\alpha\beta}$ for a smooth boundary, where $\delta_{\alpha\beta}$ is the Kronecker delta. Furthermore, $\Delta u_\beta(\mathbf{y},t)$ denotes the crack-opening-displacements (CODs) defined by $\Delta u_\beta(\mathbf{y},\tau) = u_\beta(\mathbf{y} \in \Gamma^+_C,\tau) - u_\beta(\mathbf{y} \in \Gamma^-_C,\tau)$. All integrals are understood in the sense of Cauchy principal values. The tensor $\mathbf{e}_{\varepsilon\delta}$ in Eq. (8) is the 2D permutation tensor. The relations between the unit normal vectors and the unit tangential vectors are given by

$$n^j_\alpha = \mathbf{e}_{\beta\alpha} \tau^j_\beta \;\Rightarrow\; n^j_\alpha \partial_\beta - n^j_\beta \partial_\alpha = \mathbf{e}_{\beta\alpha} \frac{\partial}{\partial s}, \qquad j = 1,2 \tag{12}$$

on the crack-faces, and by

$$n^j_\alpha = \mathbf{e}_{\alpha\beta} \tau^j_\beta \;\Rightarrow\; n^j_\alpha \partial_\beta - n^j_\beta \partial_\alpha = \mathbf{e}_{\alpha\beta} \frac{\partial}{\partial s}, \qquad j = 1,2 \tag{13}$$

on other boundaries.

3. Numerical solution procedure

To solve Eqs. (8)–(10) numerically, discretizations in both time and space with proper interpolation functions are required.

3.1. Discretization in time and space

The time interval of interest $[0,t]$ is divided into m equal steps of the span Δt, while a collocation method is used for the spatial discretization by using constant elements. Straight boundary elements of constant length are chosen to discretize the crack-faces $\sum_{i=1}^p \Gamma^i_C$, while straight boundary elements of variable length are used for the boundary Γ^0.