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On the influence of membrane inertia and shear deformation on the geometrically non-linear vibrations of open, cylindrical, laminated clamped shells

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ABSTRACT

It is the objective of this work to verify the validity of different approximations in the analysis of geometrically non-linear vibrations of open, cylindrical, laminated, fully clamped shells. Namely, it is intended to verify if membrane inertia and transverse shear deformation can be neglected and when. *p*-version finite elements with hierarchic basis functions are employed to define the models. A simple comparison of the different stiffness and mass terms is carried out first, to assess the relative importance of membrane inertia and shear deformation. Then a few numerical tests are carried out in non-linear vibrations by solving the ordinary differential equations of motion by Newmark's method. It is found that what are usually considered to be thin panels actually require the consideration of shear deformation.

1. Introduction

Laminated composite shells provide stiff, aerodynamically efficient surfaces, have good corrosion and fatigue properties, and give weight reduction in comparison to metal based structures. Thus, laminated shells encounter a wide number of applications and in some of these, e.g. in aeronautic structures, they may experience large amplitude vibrations. This work is on forced, geometrically non-linear vibrations of cylindrical, laminated, open panels and has the purpose of verifying if approximations that are frequently adopted are valid. The specific interest is in:

(1) comparing thin shell theory, that neglects shear strains, with first-order shear deformation theory (FSDT). There are a number of "rules of the thumb", namely the 1/20 relation between length and thickness, that are employed by engineers to choose between thin or FSD shell theories, but a validation of these rules is absent in non-linear vibrations;
(2) analysing the influence of the membrane inertia in the non-linear dynamics of laminated shallow shells. In fact, although it is very popular to neglect membrane inertia, it is not certain that this approximation is reasonable in shells vibrating with large amplitude displacements.

A literature review focused on the above aspects is carried out in Section 2. In Section 3 the models are presented. A simple comparative analysis is executed in Section 4 using a model with one degree of freedom per displacement component. Multi-degree-of-freedom models are employed in Section 5, where numerical tests are performed to compare the different models and discuss their validity. The numerical tests will employ a *p*-version finite element [1] with hierarchic basis functions, which was first used in [2]. Newmark's method [3] will be used to solve the differential equations of motion in the time domain. The thickness and the curvature radius will be changed in order to analyse how shear deformation and membrane inertia affect the shells.

2. Literature review

Several theories exist for the analyses of laminated shells. For introductions to these theories and general reviews on shell vibrations, review papers [4–7] and books [8–12] may be consulted. A literature review focused on studies that addressed the importance of shear deformation and membrane inertia is carried out in the next sub-sections. We are in particular interested in open, cylindrical, composite laminated, shallow shells, but will include some works on plates in this review.

In a shallow shell (Fig. 1) the raise is small in comparison with the spans, being generally accepted that the smallest radius of curvature should be lower than twice the projected length *b*, or more restrictively, $b/R \leqslant 0.4$ according to [13]. When the shell is shallow, Cartesian coordinates can be used and the strain displacement relations slightly differ from the ones of plates [10]. The analysis of this paper is restricted to cylindrical open shells, and the geometry of a shell prior to deformation is defined from a reference

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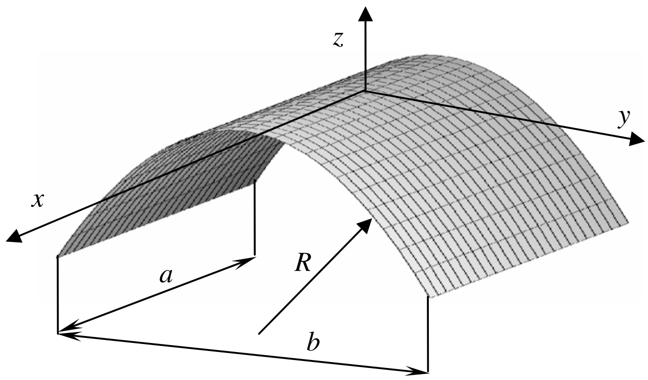

Fig. 1. Open cylindrical shell.

plane by an initial “displacement” w^1 in the z direction written as $w^1(x,y) = -y^2/(2R)$ where R is the principal curvature radius.

2.1. Effect of membrane inertia

This review starts with works on linear vibrations and in particular with the paper by Leissa and Kadi [13], where the effect of membrane inertia on the linear natural frequencies of a rather thin isotropic shallow shell was analysed. Neglecting the membrane inertia caused small errors – below 1%. In [14] boundary-domain element methods were applied. The effects of neglecting the membrane and the rotary inertias on the linear natural frequencies of lower modes of thick, very shallow shells ($h/a = 0.1$, $a/R \leqslant 0.1$) were small.

A number of studies on non-linear vibrations followed a single-mode approach or severely reduce models. Abe et al. [15] examined the non-linear free vibrations of laminated shallow shells with clamped edges, considering one or two degrees of freedom. The same authors [16] analysed the response of simply supported, symmetric laminated shallow shells using a model with three degrees of freedom. Both in [15] and [16] the membrane and rotary inertias were neglected. After a literature search, Przekop et al [17] concluded that the membrane inertia has been generally neglected in the analyses of large-amplitude free vibration of shallow shells or that, when it was considered, it was in conjunction with simple one degree of freedom models. One exception occurs in [18] where a multi-degree-of-freedom model was applied following FSDT and apparently the membrane inertia was considered; however the oscillations were assumed to be harmonic, which is an important restriction. Przekop et al [17] applied two types of modal reduction to investigate free vibrations of isotropic shells: one formulation does not neglect membrane inertia and the other uses bending modes only and neglects the membrane inertia. It was concluded that the two modal formulations can predict completely different characteristics. In [19], the effect of membrane inertia on large amplitude vibrations of a complete circular cylindrical isotropic shell was investigated and it was shown that membrane inertia should be included to have an accurate model in that case. In [20], it is considered that in shallow open panels the effect of membrane inertia should be less important than for complete circular shells, but the membrane inertia is employed in the analysis of isotropic shells. The rotary inertia and transverse shear strains were neglected in [19,20].

It is concluded from an overview of the literature, that most authors accept that the membrane inertia can be neglected in the analyses of large amplitude vibrations of open shallow shells, but reference [17] indicates that this may not be always true.

2.2. Shear deformation

The first-order shear deformation theory (FSDT) [10] has some advantages in comparison with the classical laminate thin shell theory. First of all, since the transverse shear deformation and the rotary inertia are not neglected more accurate results are generally achieved using FSDT. This becomes more significant in composite laminates, because their shear moduli are generally smaller than the shear modulus of isotropic materials. Second, FSDT finite elements only require C_0 continuity whilst the thin shell ones require C_1 continuity.

In the FSDT, the displacement components in the x, y and z directions, respectively represented by u, υ and w are given by

$$u(x,y,z,t) = u^0(x,y,t) + z\theta_y^0(x,y,t), \quad v(x,y,z,t) = v^0(x,y,t) - z\theta_x^0(x,y,t), \quad w(x,y,z,t) = w^0(x,y,t) \tag{1}$$

where u^0, υ^0 and w^0 are the values of u, υ and w at the reference surface, and θ_x^0 and θ_y^0 are the independent rotations of the normal to the middle surface about the x and y axis.

Thin shell theories follow assumptions often attributed to Love, which can be found, for example in p. 6 of Ref. [9]. Due to its relevance to this work, we recall the following assumption, known as Kirchhoff’s hypothesis: “normals to the undeformed middle surface remain straight and normal to the deformed middle surface and suffer no extension”. Transverse shear is hence neglected, and $\theta_y^0(x,y,t) = -w_{,x}^0(x,y,t)$ and $\theta_x^0(x,y,t) = w_y^0(x,y,t)$. Naturally, the thinner the panel the more accurate Kirchhoff’s hypothesis is. The effect of shear deformation depends upon the dimensions, lamination and boundary conditions [8].

In his book [8], Chia states that “the transverse shear effect on deflection, buckling load, and fundamental frequency of composite plates is generally significant at the span-to-thickness ratio of 20″. In the same book it is stated that a span-to-thickness relation approximately equal to 25 requires using theories of higher order than the thin one. In [21], Berger hypothesis and a one mode approximation were adopted to analyse the large-amplitude vibration of rectangular plates with span-to-thickness relation equal to 10 and 20. Shear deformation and rotary inertia played a considerable role in the large-amplitude vibration of orthotropic plates, causing an increase in the period. The effect of thickness upon the large amplitude vibrations of isotropic shallow shells was also investigated in [22], where membrane and rotary inertia were neglected arriving at a one degree of freedom model. It was found that thin shell theory underestimates the hardening spring effect for thick shells. In [23], a third-order expansion in the thickness coordinate for the in-plane displacements was assumed to compute the linear natural frequencies of laminated plates. The authors conclude that thin plate theory is accurate for plates where $a/h \geqslant 40$, if an average error of 5.5% is acceptable. FSDT theory provided the 10th natural frequency of plates, where $a/h = 5$ with a difference of about 3% in relation to higher order theories; smaller differences occurred in the lower modes. A higher order theory is again used in [24]; the FSDT gives fundamental frequencies that are quite close to the ones computed with higher order theories and with the theory of elasticity in plates until, at least $a/h = 5$. Thin plate theory only provided reasonably accurate fundamental frequencies for a/h above 25 (error over 5%, when $a/h = 25$). In a work carried out on isotropic beams and plates [25], it was found that FSDT and thin theory may predict differently the stability of the solutions. In Ref. [26], linear natural frequencies are computed using both thin and FSDT models. In a fully clamped rectangular plate, where $a/h = 50$, the thin theory provides the first natural frequency with an error approximately equal to 2.4% and the fourth natural frequency with an error slightly larger than 3%. When the span to thickness ratio is 25, the errors are greater than 9%.

2.3. Shear correction coefficient

When a first order shear deformation theory is followed, shear correction coefficients, λ, are required. There are a few studies