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Numerical modeling of the elastic behavior of fiber-reinforced composites with inhomogeneous interphases

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Abstract

This paper describes a numerical approach for modeling the micromechanical behavior and macroscopic properties of multiphase fiber-reinforced composites with inhomogeneous interphases. The interphases are modeled as functionally graded elastic layers with the Young's modulus and Poisson's ratio varying in the radial direction. In general, the fibers can have different elastic properties and sizes and can, if desired, be randomly distributed. The approach is based on the numerical solution of a complex boundary integral equation in which the boundary parameters are expressed in terms of complex Fourier series. All the integration can be done analytically and thus the method allows for accurate calculation of the elastic fields anywhere within the material, including inside the fibers and interphases. Explicit expressions for the effective elastic constants can be obtained from general relations between the average stresses and strains. Numerical experiments and comparisons with the numerical benchmark results available in the literature have demonstrated the versatility, accuracy, and efficiency of the presented approach.

Keywords: Fiber-reinforced composites; Inhomogeneous interphase; Micromechanical modeling; Macroscopic properties; Boundary integral equation; Complex Fourier series

1. Introduction

Fiber-reinforced composites are widely used in engineering applications due to their superior mechanical performance. In this class of materials, interphases (see Fig. 1) are often present between the fibers and the surrounding matrix due to some well-known reasons such as protective coating of the fibers and chemical reactions during the manufacturing process. Although small in thickness, the interphases have significant effects on the micromechanical behavior as well as on the overall macroscopic properties of the fiber-reinforced composites (see e.g. [1–7]). In many cases, the interphases have spatially varying rather than uniform properties. The gradation of the interphase properties causes changes in the effective properties and in the state of stress in the composites, and also affects their functionality and reliability [8–12]. Thus, the presence of inhomogeneous interphases needs to be taken into account in numerical modeling.

The effects of interphases on the micro- and macromechanical behavior of fiber-reinforced composites have been extensively studied (see reviews in [7,13,14]). Most of the studies, however, are based on the traditional effective-medium theories or on the simplifying assumptions of dilute distribution of the inclusions [10–12,15–23] and restricted to estimation of the overall properties of the material. In these analyses, the interactions between the fibers are not directly taken into account. In addition, most of the approximation schemes used to

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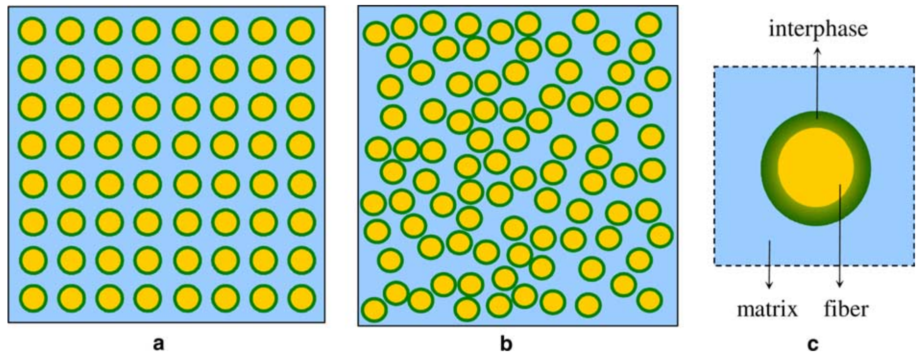

Fig. 1. A composite with (a) regularly or (b) randomly distributed fibers; (c) the individual phases.

estimate the effective properties are valid only when the concentration of the fibers is small [24].

More elaborate analyses using the finite element method have also been reported for modeling fiber-reinforced composites with homogeneous [4,5,25] and inhomogeneous [6] interphases. In finite element simulations, very fine meshes need to be used inside and around the interphase layers, resulting in large numbers of degrees of freedom. Accordingly, finite element models typically employ the concept of a unit cell with an assumed regular distribution of fibers. Even with these assumptions, however, the method is computationally intensive and the accuracy of the results depends somewhat on the mesh size.

The work on boundary element modeling of fiber-reinforced composites with interphases is rather limited, despite of the dimension advantage of this method over the finite element method. Conventional boundary element approaches suffer from drawbacks of the collocation method, which imposes practical limitations on the thickness of the interphases and the size of the problem. Recently, Liu et al. [7] presented a direct boundary element method for modeling fiber-reinforced composites with homogeneous interphase layers, where a unit cell containing only one circular fiber was modeled, and Chen and Liu [26] extended the work to include multiple cells. Yao et al. [27] incorporated the fast multipole method in a substructuring BEM to model fiber composites with homogeneous interphases. To the authors' knowledge, no work has been reported for boundary element modeling of fiber-reinforced composites with inhomogeneous interphases.

In a recent paper, Mogilevskaya and Crouch [14] presented a new approach for modeling multi-phase fiber-reinforced composites with uniform interphases. This approach was originally suggested by Mogilevskaya and Crouch [28] for fiber-reinforced composites with perfectly bonded fibers. They used two-dimensional model in which the problem was reduced to the one of an infinite isotropic elastic plane containing multiple circular elastic inclusions. This model was later extended to include homogeneously imperfect interface [29], cracks [30,31], and circular holes [32]. These analyses combine the series expansion technique with a direct boundary integral method in which the unknown boundary parameters are expressed globally in terms of series expansions of orthogonal functions (i.e., Fourier series for circular boundaries and Chebyshev polynomials for cracks). All the integrals involved in the analyses are evaluated analytically and numerical errors only come from truncation of the series. This approach was further extended to incorporate a fast multipole algorithm for large-scale problems [33] and to consider a finite rectangular domain in combination with an embedding technique [34].

This paper presents an extension of the work by Mogilevskaya and Crouch [14] to model micromechanical behavior and macroscopic properties of fiber-reinforced composites with radially graded interphases. In general, the fibers can be randomly distributed and the elastic properties of the fibers can, if desired, be arbitrary. The interphase layer for each inclusion is modeled by a system of thin bands with different properties that are constant for each band but vary from band to band to provide the piece-wise constant approximation of the function that governs the gradation of the interphase material. Similar to the previous work [14], the global approximation of the unknown boundary parameters by the truncated Fourier series and analytical integration are used in the present extension. Because of its analytic nature, the approach has advantages in accuracy and efficiency over some other numerical methods such as the finite element and boundary element methods for the particular problem under consideration. It allows one to calculate elastic fields everywhere in the matrix and the fibers including inside the interphases.

2. Computational model

As it has been done in [14], the fibers are idealized as uniform, infinite, unidirectional cylindrical inclusions