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Optimisation of sandwich panels with functionally graded core and faces

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ABSTRACT

The distributions of properties across the thickness (core) and in the plane (face sheets) that minimise the interlaminar stresses at the interface with the core are determined solving the Euler–Lagrange equations of an optimisation problem in which the membrane and transverse shear energy contributions are made stationary. The bending stiffness is maximised, while the energy due to interlaminar stresses is minimised. As structural model, a refined zig-zag model with a high-order variation of displacements is employed. Simplified, sub-optimal distributions obtainable with current manufacturing processes appear effective for reducing the critical interfacial stress concentration, as shown by the numerical applications.

1. Introduction

Sandwich structures are broadly used because they offer a high bending stiffness with the minimum mass, but also by virtue of their capability to be tailored in order to meet design requirements, their high damping properties and the great potential for impact protection, containment of explosions and projection of fragments [1] they offer. Unfortunately they suffer from strong stress concentrations at the interfaces among the face sheets, the weak adhesive layer and the core, as a consequence of the distinctly different properties of these materials in contact. It represents a critical design concern since it can have detrimental effects on structural performance and service life and can cause a premature failure at load levels much lower than the ultimate load. A number of sophisticate models has been recently published in order to accurately and efficiently predict these potentially deleterious effects, whose discussion is outside the purpose of this paper (see, e.g., the comprehensive review paper by Noor et al. [2] for details). It is just remarked that, in order to accurately predict the stress fields, the sandwich structural models have to account for the core deformability under compression and shear, as well as for the through-the-thickness shear and normal stresses continuity at the face–core interfaces necessary for keeping equilibrium. The recent fast growing interest to new technologies has provided an excellent opportunity for refining these models and, in general, the studies on sandwich composites. Some high-order models for the analysis of sandwich composites can be found in Li et al. [3], Li and Kardomateas [4], Frostig [5] and Schwarts-Givli et al. [6].

New perspectives of solution for the stress concentration problem are represented by nano-technologies (see, e.g., Mahfuz et al. [7]) and functionally graded materials – FGM – (see, e.g., Suresh and Mortensen [8] and Fuchiyama and Noda [9]), because they enable either development of new high performance materials with improved damage tolerance, or a smooth property variation that can optimise some structural functions. Moreover, Aliaga and Reddy [10] proposed a study on FGM by a third-order plate theory. Although these new technologies are in their infancy, their potential advantages appear great. In the immediate future, the best chance for fully exploiting the lightweighting potential of sandwich composites appears to be the tailoring optimisation of the face sheets combined with a functionally graded core. To minimise the change in stiffness and then the interlaminar stress concentration at the interface, the core should have smoothly property variations across the thickness, that at this position are similar to those of the faces (see, e.g., Apetre et al. [11]).

While the FGM do not complicate so much the simulation, or even make it much easier by virtue of their smooth property variation, they obviously represent manufacturing complications. The methods till now proposed have been developed for thermal coatings and thermal protection systems (see, e.g., [12–16]), but it is a common opinion that the technology is mature for the production of core foams with a desired variation of properties.

Based on this consideration, recently, a series of papers has been presented by Sankar and Tzeng [17], Sankar, [18], Venkataraman and Sankar [19] and Apetre et al. [20] about FG laminated and sandwich beams with FG cores. The materials were assumed to be isotropic and with an exponential variation of the elastic

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stiffness coefficients across the thickness, in order to allow exact elasticity solution via Fourier transform methods. Polynomial distributions were considered by Apetre et al. [11,21] and Zhu and Sankar [22], who used the Galerkin method. These studies focusing on the capability of FGM to reduce the stress concentrations encourage both the development of new methods for the production of foam core materials with FG properties and further theoretical developments in order to understand which property distribution achieves the wanted properties.

In the present paper, a numerical study of the potential advantages of combining tailoring optimisation and FGM is presented. A new optimisation technique recently developed by the authors [23] is employed for finding the orientation of the reinforcement fibres of the face sheet layers and the law of variation of the core properties across the thickness, able to minimise the interlaminar stress concentration. A refined zig-zag model is employed as structural model, in order to accurately and efficiently account for the interfacial stress continuity conditions. To make the structural model more efficient, a strain energy updating procedure is employed, like in Ref. [24]. The basic idea is to carry out the analysis with a classical C° parent finite element model based on the First-Order Shear Deformation Plate Theory-FSDPT and to update its energy to that of the zig-zag model in the post-processing phase, where the stresses are computed.

2. Structural model

2.1. Kinematics

Assume the sandwich panel to consist of $\mathcal{S}$ layers with different thickness and material properties, the core being treated as a thick layer in a multilayer construction (the accuracy of this hypothesis will be assessed in Section 4). In order to accurately capture the stresses fields across the thickness, the whole set of displacement and stress contact conditions has to be fulfilled at the interfaces of the constituent layers. To this purpose, the displacement field is represented as

$$U(x,y,z) = u(x,y,z) + \mathcal{U}(x,y,z) \tag{1a}$$
$$V(x,y,z) = v(x,y,z) + \mathcal{V}(x,y,z) \tag{1b}$$
$$W(x,y,z) = f_w^1(x,y,z) + f_w^2(x,y,z) \tag{1c}$$

according to Ref. [23]. Like for equivalent single-layer models, u and v give contributions to in-plane displacements that are continuous together with their first derivatives across the thickness

$$u(x,y,z) = u^{(0)} + z(\gamma_x^{(0)} - w_{,x}^{(0)}) + z^2\mathcal{C}_x(x,y) + z^3\mathcal{D}_x(x,y) \tag{2a}$$
$$v(x,y,z) = v^{(0)} + z(\gamma_y^{(0)} - w_{,y}^{(0)}) + z^2\mathcal{C}_y(x,y) + z^3\mathcal{D}_y(x,y) \tag{2b}$$

while

$$\mathcal{U}(x,y,z) = \sum_{k=1}^{\mathcal{S}-1} {}^{(k)}\phi_x(x,y)(z-{}^{(k)}Z^+)\mathcal{H}_k \tag{3a}$$
$$\mathcal{V}(x,y,z) = \sum_{k=1}^{\mathcal{S}-1} {}^{(k)}\phi_y(x,y)(z-{}^{(k)}Z^+)\mathcal{H}_k \tag{3b}$$

are continuous, but have discontinuous first derivatives at the interfaces. The two contributions to the transverse displacement have the following explicit expressions

$$f_w^1(x,y,z) = a(x,y) \tag{4a}$$
$$f_w^2(x,y,z) = zb(x,y) + z^2c(x,y) + z^3d(x,y) + z^4e(x,y) + \sum_{k=1}^{\mathcal{S}-1} {}^{(k)}\psi_x(x,y)(z-{}^{(k)}Z^+)\mathcal{H}_k + \sum_{k=1}^{\mathcal{S}-1} {}^{(k)}\psi_y(x,y)(z-{}^{(k)}Z^+)^2\mathcal{H}_{,k} \tag{4b}$$

a being assumed as the transverse displacement on the reference mid-plane $w^{(o)}$. Also in this case, a field with discontinuous derivatives is superposed to a polynomial representation. The continuity functions ${}^{(k)}\Phi_x$, ${}^{(k)}\Phi_y$, ${}^{(k)}\psi_x$ and ${}^{(k)}\psi_y$ are determined enforcing the continuity of the transverse shear and normal stresses σ_{xz}, σ_{yz}, σ_{zz} and of the gradient $\sigma_{zz,z}$ at the interfaces, as prescribed by the elasticity theory. The functions $\mathcal{C}_x$, $\mathcal{C}_y$, $\mathcal{D}_x$, $\mathcal{D}_y$, b, c, d, e are determined enforcing the fulfilment of the boundary conditions at the upper and lowers faces.

Note that the functional degrees of freedom $u^{(o)}$, $v^{(o)}$, $w^{(o)}$, γ_x and γ_y coincide with those of conventional equivalent single-layer plate models. This will enable the possibility to update the strain energy of these models to that of the present model, with the purpose and the technique described hereafter. The zig-zag model is used in the optimisation process of Section 3, while the analysis of the optimised solutions is carried out as outlined in the following section.

2.2. Energy updating

Unfortunately, a direct finite element formulation by the zig-zag model of Eqs. (1)–(4) involves second order derivatives of the nodal d.o.f. that make the element inefficient. Since, a finite element approach is required for treating the spatial variation of the mechanical properties set by the optimisation process, the problem is here overcame updating the strain energy of a C° parent finite element model based on the FSDPT, through the technique of Ref. [24]. This element is used for a preliminary analysis, then its energy is updated in the post-processing phase to that of the zig-zag model. As a result of this process, corrective terms for the d.o.f. of the FSDPT are computed and used for the stresses analysis. A broad summary of this technique is reported hereafter.

The first operation is the interpolation and smoothening of the results by the FSDPT finite element analysis at discrete points, in order to compute the derivatives from the interpolation instead from the representation (with a lower order) by the shape functions. For this operation, spline functions are used. In this phase, the kinematics of the FSDPT model is made consistent with that of the zig-zag model, i.e. $\theta_x = z(\gamma_x^{(0)} - w_{,x}^{(0)})$, $\theta_y = z(\gamma_y^{(0)} - w_{,y}^{(0)})$. Assume $u^{(o)}$, $v^{(o)}$, $w^{(o)}$, γ_x and γ_y as the functional d.o.f. of the FSDPT model, while the homologous terms of the zig-zag model be indicated by the superscript $()^{\sim}$. All the updating operations be carried out using corrective terms, i.e.

$$\tilde{u}^{(0)} = \hat{u}^{(0)} + \Delta\hat{u}^{(0)} \tag{5a}$$
$$\tilde{v}^{(0)} = \hat{v}^{(0)} + \Delta\hat{v}^{(0)} \tag{5b}$$
$$\tilde{w}^{(0)} = \hat{w}^{(0)} + \Delta\hat{w}^{(0)} \tag{5c}$$
$$\tilde{\gamma}_x^{(0)} = \hat{\gamma}_x^{(0)} + \Delta\hat{\gamma}_x^{(0)} \tag{5d}$$
$$\tilde{\gamma}_y^{(0)} = \hat{\gamma}_y^{(0)} + \Delta\hat{\gamma}_y^{(0)} \tag{5e}$$

It is remarked that $u^{(o)}$, $v^{(o)}$, $w^{(o)}$, γ_x and γ_y are computed by the FSDPT finite element model, while the corrective terms are unknown quantities to compute equating the energy contributions of interest by the penalty function method. The continuity functions ${}^{(k)}\Phi_x$, ${}^{(k)}\Phi_y$ are preliminarily computed in an approximate way assuming $w = w^{(o)}$, i.e. $f_w^2 = 0$, then they are made consistent with the present zig-zag model making the transverse shear strains $\varepsilon_{xz}\varepsilon_{yz}$ least square compatible with their consistent counterparts. This is numerically more efficient than calculating them directly from the contact condition.

The updating of the transverse shear energy is performed equating these two homologous quantities for the zig-zag and FSDPT models

$$(\boldsymbol{q_e} + \Delta\boldsymbol{q_{eK}})^T\boldsymbol{K}_{\mathbf{fsdpt}}(\boldsymbol{q_e} + \Delta\boldsymbol{q_{eK}}) = \boldsymbol{q_e}^T\boldsymbol{K}_{\textbf{zig-zag}}\boldsymbol{q_e} \tag{6}$$

where $\boldsymbol{q_e}$ represents the vector of nodal d.o.f., $\Delta\boldsymbol{q_{eK}}$ the corrective terms and $\boldsymbol{K}$ is the stiffness matrix (only the rows and columns rel-