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Composites Science and Technology 67 (2007) 2683–2690

COMPOSITES SCIENCE AND TECHNOLOGY

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On contact problem of an elastic laminated half-plane with a boundary normal to layering

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Received 8 December 2006; received in revised form 26 February 2007; accepted 26 February 2007
Available online 2 March 2007

Abstract

This paper deals with the plane contact problem of an infinite long cylinder pressured into an elastic laminated semi-space. The non-homogeneous body is composed of periodically repeated two constituent laminae and its boundary is assumed to be normal to the layering. The homogenized model with microlocal parameters given by Woźniak [Woźniak C. A nonstandard method of modelling of thermoelastic periodic composites. Int J Eng Sci 1987;25:483–99]. Matysiak and Woźniak [Matysiak SJ, Woźniak C. Micromorphic effects in a modeling of periodic multilayered elastic composites. Int J Eng Sci 1987;25:549–59.] are applied to find an approximate solution to the problem. The problem is reduced to well-known dual integral equations and the stresses are obtained in the form of Fourier integrals. Numerical results, which show the influence of geometrical and mechanical properties of composite constituents on stress distributions, are presented in figures.

Keywords: C. Laminate theory; C. Elastic properties; C. Stress concentration; C. Deformation

1. Introduction

The knowledge of stress distributions in contact to two bodies play an important role in many technological applications. For this reason, considerable progress has been made with the analysis of contact problems in the theory of elasticity (see, for example monographs [6–11,15,22,23,26,28]). Some contact problems of anisotropic bodies were investigated by Wills [32], Hwu and Fan [13], Rogowski [24,25], and Liao [20].

Composite materials with periodic structures are widely utilized in building engineering, machine elements, and aviation structures. Some composite structural members are constructed to work in contact with others. The two-dimensional contact problems of periodically layered half-space with the layering parallel to the boundary were considered by Kaczyński and Matysiak [16–18]. The above-mentioned papers are based on the homogenized model with microlocal parameters given by Woźniak [33] and in the case of elastic laminates by Matysiak and Woźniak [21]. This model was derived by using the concepts of the nonstandard analysis combined with some postulated a priori physical assumptions. The governing equations of the homogenized model are expressed in terms of unknown macro-displacements and certain extra unknowns called microlocal parameters and they replace the partial differential equations with strongly oscillating coefficients of the theory of elasticity. It should be emphasized that the continuity conditions on interfaces are fulfilled within the framework of the homogenized model.

This paper deals with the two-dimensional contact problem of a periodically two-layered elastic half-space with an infinitely long, rigid cylinder. The boundary of the composite is assumed to be normal to the layering, and the radius of the pressured cylinder is much greater than the width of contact region. The considered problem will be solved within the framework of the homogenized model with

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doi:10.1016/j.compscitech.2007.02.013

microlocal parameters (see [21,33]). Moreover, the boundary condition describing the stress component normal to the boundary plane, discontinuous on interfaces is replaced by some averaging condition. This approach allows to reduce the considered mixed boundary problem to dual integral equations and to determine the contact pressures, the width of contact region, and displacements, stresses in the composite half-space. It is not possible to get an exact, closed form of the solution of the contact problem, so this paper presents some approximate solution (the exact closed form solution within the framework of the homogenized model with microlocal parameters). Other approximation models are known. They were derived by using some averaging procedures: the classical asymptotic homogenization [2,14,27,35]; the procedure based of the concept applied in the theory of thick plates (cf. [1]); the procedure based on the mixture theory [3]; the procedure based on some physical assumptions [12] or on the matrix method [4,5] or on the tolerance averaged procedures proposed by Woźniak and Wierzbicki [34]. However, the homogenized model with microlocal parameters seems to the useful to describe the contact problem, because: (1) the model leads to two partial differential equations with constant coefficients in the two-dimensional case, (2) the continuity conditions on interfaces are satisfied, and (3) many boundary value problems for periodically layered composites have been solved within the framework of the model. Moreover, the substitution of the averaging boundary condition and the application of homogenized model with microlocal parameters leads to a good approximation of stresses for some problems of a semi-infinite laminated layer (see [19] where the results obtained within the framework of the homogenized model and the classical elasticity are compared).

2. Formulation and solution of the problem

Let us consider a microperiodic laminated half-space, the middle cross-section of which is shown in Fig. 1. A repeated fundamental layer (lamina) of thickness δ is composed of two homogeneous isotropic elastic sublayers, with thicknesses δ_1 and δ_2. They are characterized by Lamé constants λ_j, μ_j, $j = 1, 2$.

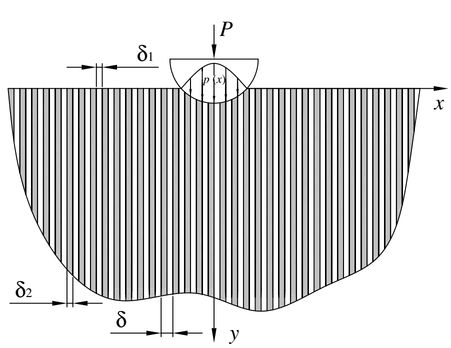

Fig. 1. The scheme of considered problem.

The boundary plane of half-space is assumed to be normal to the layering. Perfect bonding between the layers is assumed. A Cartesian coordinate system (x, y, z) such that the y axis is parallel to the layering and the x axis coincides with the straight line normal to the layering is introduced. Restricting the considerations to the plane-strain state (independent on the variable z), denote at the point $(x, y, 0)$ the displacement vector by $[u(x, y), v(x, y), 0]$ and the stresses by $\sigma_{xx}^{(j)}, \sigma_{xy}^{(j)}, \sigma_{yy}^{(j)}, \sigma_{zz}^{(j)}$, $j = 1, 2$ (in the following, all quantities pertaining to the sublayers of the first kind will be denoted with the index $j = 1$ and of the second kind with $j = 2$).

Consider an infinitely long rigid cylinder pressured into the laminated half-space, Fig. 1. The cross-section of punch is assumed to be in the shape of a parabola (it can be used to the approximation of a circular punch with large radius R):

$$g(x) = \frac{x^2}{2R}, \quad R = \text{const.} > 0. \tag{2.1}$$

The problem of indentation of the periodically two-layered half-space is frictionless and has an unknown width of contact zone.

The basis of considerations is the homogenized model with microlocal parameters (see [21,33]). Such an approach has proved to be useful and effective for solving a variety of boundary problems (see [36]). Below we recall below some relevant results from this theory. The homogenized model with microlocal parameters in the case of plane-strain state for the stratified body can be determined by following relations (see for example [16–18]):

$$\begin{aligned}
&u(x, y) = U(x, y) + h(x)q_x(x, y) \cong U(x, y),\\
&v(x, y) = V(x, y) + h(x)q_y(x, y) \cong V(x, y),\\
&\sigma_{yy}^{(j)}(x, y) \cong (\lambda_j + 2\mu_j)V_{,y} + \lambda_j(U_{,x} + h_j q_x),\\
&\sigma_{xx}^{(j)}(x, y) \cong (\lambda_j + 2\mu_j)(U_{,x} + h_j q_x) + \lambda_j V_{,y},\\
&\sigma_{xy}^{(j)}(x, y) \cong \mu_j(U_{,y} + V_{,x} + h_j q_y), \quad j = 1, 2,
\end{aligned} \tag{2.2}$$

where U, V and q_x, q_y are unknown functions interpreted as the macro-displacement and microlocal parameters, respectively, and $h(x)$ is a priori given δ – periodic function (called the shape function) defined as follows:

$$h(x) = \begin{cases} x - 0.5\delta_1, & 0 \leqslant x \leqslant \delta_1, \\ \frac{-\eta x}{1-\eta} - 0.5\delta_1 + \frac{\delta_1}{1-\eta}, & \delta_1 \leqslant x \leqslant \delta, \end{cases} \quad h(x + \delta) = h(x), \tag{2.3}$$

and

$$\eta = \delta_1/\delta. \tag{2.4}$$

The shape function given by Eq. (2.3) is chosen in such way, that the continuity conditions for the stress vector on the interfaces are satisfied. The symbol h_j, $j = 1,2$, denotes the derivative of the function $h(x)$ in the jth kind of composite component and

$$h_1 = 1, h_2 = -\frac{\eta}{1-\eta}. \tag{2.5}$$