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Reducing tensile stress concentration in perforated hybrid laminate by genetic algorithm

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Abstract

Holes are a common geometric discontinuity in composite structures. Since stress concentrations at a hole can decrease strength, it is desirable that such concentrations be as low as possible. This paper demonstrates ability to minimize tensile stress concentrations in a perforated laminated composite. Synergizing a genetic algorithm optimization method with a specially developed FEM program enables one to optimally orient the fibers locally both within the plies and from ply-to-ply, and thereby significantly reduced the tensile stress concentration associated with a hole. The present evolutionary technique provides more than just a favorable stacking sequence of various rectilinearly orthotropic plies having different fiber orientations.

Keywords: A. Structural materials; C. Stress concentrations; C. Laminates; Optimization; Mimicry

1. Introduction

Motivated by their favorable responses, composites find widespread utilization. One often employs laminated composites consisting of stacked individual rectilinearly orthotropic plies each having different fiber orientation. However, potential strength degradation associated with stress concentrations at geometric discontinuities is an important design consideration. Stress concentrations can sometimes be reduced by altering the shape of the hole or notch, or by changing the structural stiffness in the neighborhood of the discontinuity either by locally changing fiber orientation or by modifying/removing material [1–5]. Hyer and Charette decreased stress concentration and enhanced performance of a perforated composite by aligning the fibers with the principal stress directions [1,2]. Cho and Rowlands recently combined an optimization scheme with a FEA to orientate the fibers locally and thereby reduce the tensile stress concentration factor (TSCF) at a hole in a single-ply homogeneous orthotropic plate [3]. While analytical/numerical research on changing local fiber orientation seems to date to be limited to a single-ply laminate, the present analysis extends such concepts to layered composites. The TSCF is minimized here by locally optimizing the fiber orientation locally both from ply-to-ply and within each ply of the laminate. Compared with other available studies, superior results are obtained by parallel computing between the FEA and a developed genetic algorithm. This new capability is compatible with manufacturing advances [5,6]. Conventional design philosophies for composite laminates often involve changing the stacking sequence and/or thickness of the various plies [7–9]. Most engineering optimization algorithms use gradient-based methods where the search direction is a function of the gradient of design variables [10–13]. Gradient-based methods are unable to deal efficiently with complex design space or a large number of design variables [9,12–15]. These drawbacks can be overcome by using genetic algorithm (GA) optimization. Genetic-based optimization

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procedures also converge well to the global optimum value with respect to both discrete and continuous design variables. Several researchers have recently recommended GAs for optimizing composite [16–20]. This study merges a nonlinear FEA and GA with a parallel computing scheme to minimize the TSCF in a perforated tensile composite laminate by controlling fiber directions of discrete elements and individual plies.

A general nonlinear 3D, 20-node isoparametric laminated composite FEA code was specially developed for the present analysis. The element has three DOF (the three displacements) per node. The FEA code accounts for the various fiber angles in formulating the stiffness matrix. Data from the FEA are exchanged, back and forth, with the parallel processing capability of the GA optimization module, Fig. 1. Mathematical formulae of the nonlinear static FEA are presented in the next section, including a brief formulation of the nonlinear equilibrium equation of motion for FEA.

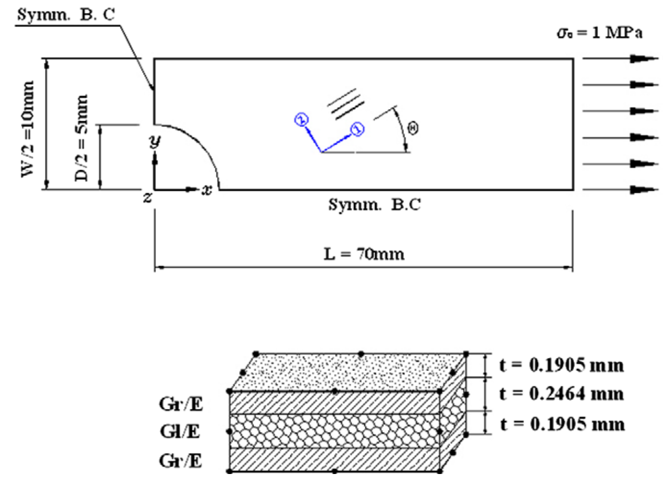

Fig. 2. Geometry model and boundary conditions for FEA of three-ply composite laminate.

2. 3D constitutive law

When representing the configuration of the solid in Fig. 2 in terms of the finite element equilibrium equation, the virtual work principle must be applied to the deformable solid under arbitrary static equilibrium condition at time t. The external virtual work at time t, i.e., ${}^tR$, can be expressed as

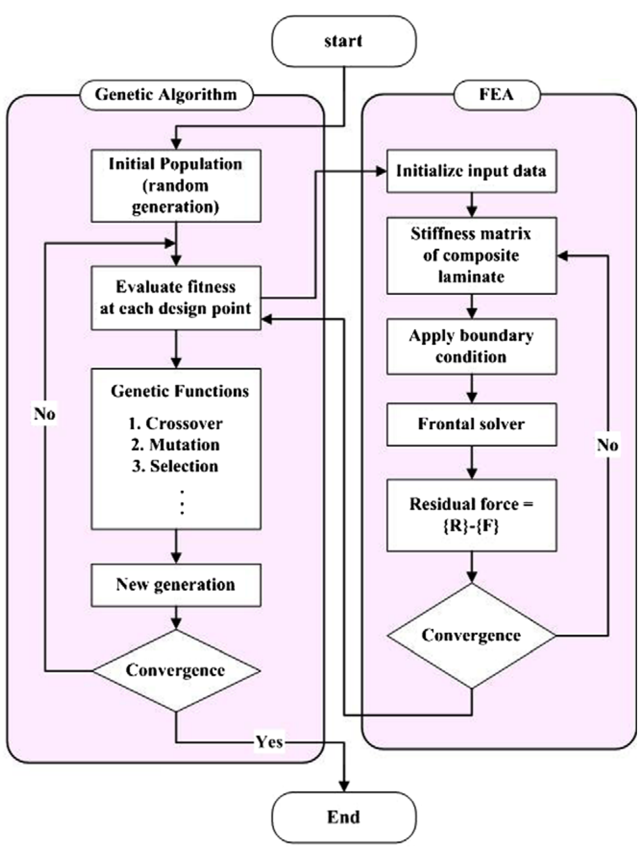

Fig. 1. Parallel computing scheme for GA and FEA.

$$\int_{^0V} {}^t_0S_{ij}\,\delta^t_0\varepsilon_{ij}\,\mathrm{d}(^0V) = {}^tR \tag{1}$$

where ${}^t_0S_{ij}$ is the 2nd Piola–Kirchhoff stress tensor or nominal (engineering) stress and ${}^t_0\varepsilon_{ij}$ is strain [21]. According to this stress definition, the real force applied to the deformed body has been transformed to the initial state and divided by initial area. The relationship between the 2nd Piola–Kirchhoff stress tensor and the Green–Lagrange strain tensor, ${}^t_0\varepsilon_{ij}$, is given by

$${}^t_0S_{ij} = {}^t_0C_{ijrs}\,{}^t_0\varepsilon_{ij} \tag{2}$$

such that ${}^t_0C_{ijrs}$ represents the material properties tensor. For a linear elastic isotropy, the material tensor can be written as

$${}^t_0C_{ijrs} = \lambda\delta_{ij}\delta_{rs} + \mu(\delta_{ir}\delta_{js} + \delta_{is}\delta_{jr}) \tag{3}$$

Quantities λ and μ are the Lame constants, $\lambda = E/(1+v)(1-2v)$ and $\mu = E/2(1+v)$, and δ_{ij} is the Kronecker delta. The matrix of the C_{ijrs} is found in reference [22].

3. Nonlinear equilibrium equation

The Green–Lagrange strain tensor at time t, ${}^t_0\varepsilon_{ij}$ can be divided into the following linear, e_{ij}, and nonlinear, η_{ij}, terms

$$\begin{aligned} {}^t_0\varepsilon_{ij} &= {}^t_0e_{ij} + {}^t_0\eta_{ij} \\ {}^t_0e_{ij} &= \frac{1}{2}({}^t_0\eta_{i,j} + {}^t_0\eta_{j,i}) = [{}^t_0B_{L0}]\{{}^t_0u\} \\ {}^t_0\eta_{ij} &= \frac{1}{2}({}^t_0u_{k,i}\cdot{}^t_0u_{k,j}) = [{}^t_0B_{L2}]\{{}_0{}^tu\} \end{aligned} \tag{4}$$

By substituting Eq. (4) into Eq. (1) and linearizing the resulting equation, it is possible to rewrite Eq. (1) as follows: