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On nonlinear behavior in brittle heterogeneous materials ☆

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Abstract

Many modern fiber-reinforced composite materials are ‘brittle’, in the sense that their strain to failure under quasi-static loading is typically of the order of 1% when loaded in directions generally controlled by fiber fracture, and the energy-to-failure under the quasi-static loading curve is typically small. For this reason, analysis of these materials is typically done under assumptions of linear elasticity, usually for homogeneous materials or material layers in a laminate. This is in contrast to ‘ductile’ metal behavior in which elastic–plastic behavior is widely discussed. What is most remarkable is the fact that for long-term behavior, the situation is nearly reversed in many cases. For design-level stresses, nonlinear behavior in ductile materials is often minor (even high temperature structures are often designed with linear analysis) but heterogeneous brittle materials may show changes of the order of 10–30% in stiffness and strength for long-term behavior under design loads that are quite safe. One cannot properly describe or model the behavior of composite laminates, therefore, without an understanding and representation of nonlinear behavior.

The present paper will focus on some of these nonlinear behaviors, and will present some recent interpretations of them, for both long-term and quasi-static loading of continuous fiber reinforced composite laminates. The paper begins with a discussion of the effect of temperature on fiber-direction stiffness of unidirectional carbon reinforced PPS composite laminates. Micromechanical models are used to estimate the changes in stiffness, strength, and life. Quasi-static and cyclic behavior of woven laminates is then considered. Data and predictions are presented.

Keywords: Composites; Brittle materials; Nonlinear behavior; Anisotropic

1. Nonlinear temperature dependent stiffness of polymers and unidirectional polymer composites

For a given loading rate, the stiffness of a polymer is known to be dependent on temperature. Arrhenius activation models are typically found to apply to such materials in the glassy state, and the WLF equation is an example of a typical relationship for polymers in the region of the glass transition temperature [1]. For robust engineering models of strength, stiffness, and life, it is necessary to have explicit relationships between stiffness and temperature over the full range of expected engineering service use, not just over a limited range of behavior. In some applications, that requires stiffness–temperature relationships that can be used over ranges of temperature that span glassy to rubbery (and even flow) behavior.

For this discussion, the stiffness variation has been divided into three regions, based on a perception that there are three distinct physical phenomena that control the transitions. In order to construct a model that spans the entire range of behavior, we observe that all of the processes that control the transitions are driven by temperature, and that all of the transitions are, in some sense, the result of changes in mobility of the macromolecules involved at the local level. Increasing the local energy in the macromolecule by

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increasing the temperature triggers changes in that mobility. Using the concept of relaxation times for each of those processes, and adopting the notation common to recent discussions of this subject, (cf. Nielsen and Landel [2]), we could write an equation for the stiffness variations in the form

$$E(t) = \sum_{i=1}^{N} E_i \exp\left[\frac{-t}{\tau_i}\right] \tag{1}$$

where t_i are the relaxation times for the different transition processes. Alternatively, we can express the relationship in integral form using a distribution function, $H(t)$, to obtain

$$E(t) = \int_{-\infty}^{+\infty} H(\tau) \exp\left[\frac{-t}{\tau}\right] \mathrm{d}(\ln(\tau)) \tag{2}$$

in which $H(\tau) = \tau E(\tau)$. The distribution functions are determined experimentally, from relaxation data according to the relationship $H(\tau) = -\frac{\mathrm{d}(E_\mathrm{r}(t))}{\mathrm{d}(\ln(t))}$, where E_r is the experimental relaxation modulus, or from retardation spectra in a creep experiment, using the companion relationship $L(\tau) = \frac{\mathrm{d}(J(\tau))}{\mathrm{d}(\ln(t))}$, where $J(\tau)$ is the shear creep compliance [2]. Since we wish to represent the modulus across all transitions, we choose a general form for the distribution function that can be used to represent data associated with all processes that may drive the transition behavior. An allowable form is

$$H_k^t(\tau) = \exp\left[-\left(\frac{t}{\tau_k}\right)^{m_k}\right] \tag{3}$$

where we have written the distribution function for the kth transition process. We assume that for a given process one can construct some relationship between time and temperature, as discussed in the previous section, and recast (3) in the form

$$H_k^t(\tau) = H_k^0 \exp\left[-\left(\frac{T}{\beta_k}\right)^{m_k}\right] \tag{4}$$

where the details of the time–temperature relationship have been absorbed into the coefficient H_k^0. Then returning to the form of (1), and considering only the three regions (and transitions) shown in Fig. 1, we use the following equation to represent the stiffness variation across the entire range of service temperature:

$$E = (E_1 - E_2)\exp\left[-\left(\frac{T}{T_1}\right)^{m_1}\right] + (E_2 - E_3)\exp\left[-\left(\frac{T}{T_2}\right)^{m_2}\right] + E_3 \exp\left[-\left(\frac{T}{T_3}\right)^{m_3}\right] \tag{5}$$

The interpretation of the material parameters in (5) is illustrated in Fig. 1.

The T_i correspond to the temperatures at each transition (as given by the maximum of the peaks on the tangent delta versus temperature curve from a DMA characterization or by the inflection point of a DSC plot) in degrees Kelvin. The values of E_i are also determined from independent experiments. E_1 is the instantaneous modulus in the glassy region of behavior, E_2 is the instantaneous modulus after the beta transition, and E_3 is the instantaneous stiffness at the beginning of the rubbery region. Van Krevelen (1990) discusses ultrasonic measurement methods for these stiffness values [3]. The dependence of these parameters on the nature of a given polymer is the subject of much literature. Intrinsic characteristics such as cross-link density, crystallinity, molecular weight, chemical structure, and density are primary factors, but extrinsic parameters such as strain, strain rate (or frequency), moisture, time and temperature also influence their values. Mahieux et al. discusses the influence of these parameters on the present model [4].

We can use the foregoing to estimate the stiffness and strength of unidirectional composites. A simple rule of mixtures illustrates this effect. Following the work of Mahieux et al. [4], if the coupling between the fiber and matrix is represented by an efficiency factor (as described by Subramanian [5]), the fiber-direction stiffness of the composite can be written explicitly in terms of temperature in the form

$$E_{11}(T) = \frac{E_m(T)V_m + \sum_{i=1}^{N}\lambda_i \exp\left[-\left(\frac{T}{T_i}\right)^{m_i}\right]E_f V_f}{V_m + \sum_{i=1}^{N}\lambda_i \exp\left[-\left(\frac{T}{T_i}\right)^{m_i}\right]V_f} \tag{6}$$

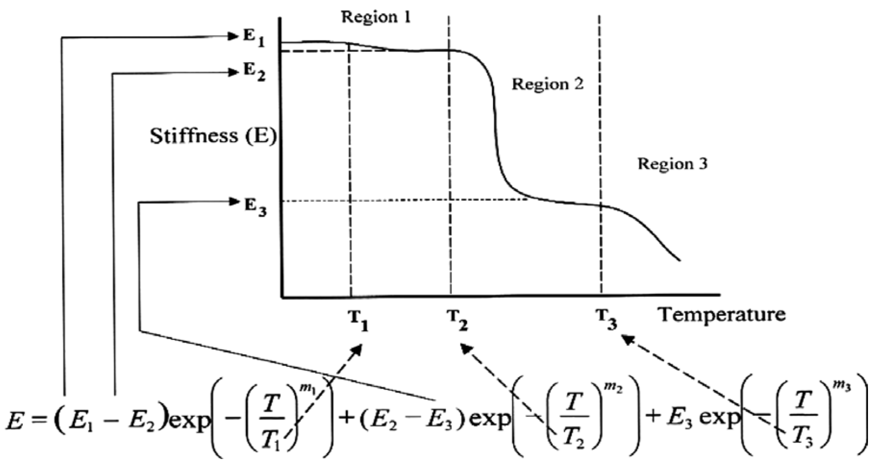

Fig. 1. Interpretation of the material parameters in (1), in terms of the transitions shown.