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On the use of quasi-dynamic modeling for composite material structures: Analysis of adhesively bonded joints with midplane asymmetry and transverse shear deformation

Joshua Radice [a,*], Jack Vinson [b]

[a] Department of Mechanical, Engineering, University of Michigan, 2203 G. G. Brown Laboratory, 2250 Hayward Street, Ann Arbor, MI 48109, United States

[b] Department of Mechanical, Engineering, University of Delaware, 126 Spencer Lab, Newark, DE 19716, United States

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Abstract

Adhesively bonded joints offer an attractive alternative to bolted/riveted approaches to joining load bearing elements because of potential weight savings and enhanced joint strength efficiency. However, modeling of these joints including substrate asymmetry, anisotropy, thickness variation, transverse shear deformation, hygrothermal growth, and complicated models for the adhesive layer can prove to be very unwieldy. This paper presents a solution methodology to model adhesively bonded joints which is capable of handling these important effects.

The differential equations of motion are modified to include fictional damping type terms. Using a modified form of the thin structural displacements including temporal variation, the quasi-dynamic governing equations for thin plates are developed. The plate relationships are simplified to yield the quasi-dynamic beam/rod relationships. An assortment of boundary and initial conditions in terms of displacements are included for completeness and to facilitate implementation. To acquire quantitative solutions, the quasi-dynamic relationships are recast into the form of finite difference equations. A form suitable for the solution of a wide range of practical problems is presented. For completeness, the boundary and initial conditions are also recast into finite difference form.

Justification of the choice to include the fictional damping terms and the discrete solution methodology is presented. The discrete governing equations are verified using the classical cantilevered beam case. Deviations from the classical solution resulting from the inclusion of transverse shear deformation and solution convergence are discussed. Once the quasi-dynamic solution methodology is validated, the single lap shear joint is modeled.

Finally, a parametric study is performed to determine the influence of asymmetric adherend stiffness property variations on the peel and shear stresses in the adhesive layer. It is observed that decreases in adherend lumped shear stiffness results in reductions in the adhesive layer stress peaks. It is also observed that asymmetrically constructed adherends are of potential benefit to the reduction of adhesive peel stresses.

Keywords: A. Adhesive joints; B. Modelling; C. Computational simulation; C. Anisotropy; B. Strength

1. Introduction

Adhesively bonded structures offer an attractive alternative to the historically prevalent bolted/riveted approaches to joining load bearing elements. Adhesively bonded assemblies allow for a gradual transfer of load from one structural element to another and a reduction of stress concentrations arising from material discontinuities inherent to mechanical fastening methods. In their respective literature reviews, Vinson [1] and Kuno [2] listed some of the other benefits of adhesively bonded construction as: weight savings from thinner adherends, improved strength to weight ratios, improved fatigue life expectancies, vibration

* Corresponding author. Tel.: +1 734 764 9156.
E-mail address: radice@umich.edu (J. Radice).

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isolation and damping, accommodation of thermal expansion mismatch, accommodation of hygrothermal swelling, improved manufacturability, reduced tooling costs, reduced machining costs, improved aesthetic appearance, improved aerodynamic efficiency, and enhanced electrical insulation capabilities.

Efficient design begins with accurate analysis. But analysis of even the simplest of symmetric isotropic joints is a formidable mathematical challenge. The difficulties are quickly exacerbated by aspects such as: asymmetric adherend architecture, non-homogeneity, and functional gradation over the length of the joint. A large portion of the composite material structures literature deals with efforts to apply these distinctly "un-metallic" properties for the improvement of load bearing capabilities and stiffness characteristics for the whole spectrum of structural applications. It seems that the tradeoff for implementation of composite structures (other than economic) is mathematical complexity versus enhanced performance capabilities.

The combination of performance advantages and analytical complexity has made adhesively bonded structures a fertile area of research. The key milestones in this field have been cited in literature reviews by Vinson [1], Kutcha and Hofer [3], Matthews [4], and many others. There have been many analytical, finite element, and experimental studies offered over the years.

Many of the published works have revolved around accurate modeling of the adhesive layers. Despite the fact that most adhesive materials are homogeneous and isotropic, the stress and deformation state in adhesive layers are known to be very complex. One of the earliest analytical attempts to capture these behaviors was by Volkersen [5]. In this study, isotropic beam adherends were analyzed using a continuously distributed shear spring model for the adhesive layer. This approach has the advantage of simplicity, but it is unable to capture the through thickness tensile stress state, or peel stresses. This is a key limitation, as the peel stress has been experimentally demonstrated to be the dominating contributor to the failure of bonded structures.

The peel stress field was first captured in the canonical work by Goland and Reissner [6]. In this work the adhesive layer is modeled as a combination of distributed shear springs and through-thickness tensile springs in a manner analogous to an elastic foundation. The limitation of this approach is identified very crisply by Frostig et al. [7] as: "the inability to control the boundary conditions of the adhesive layer and the assumption that the stresses do not vary through the thickness". Despite these drawbacks, Mortensen and Thomsen [8], Vinson [9] and many others continue to use this "workhorse" model for the adhesive layer.

This common approach was bolstered when Frostig et al. [7] performed comparative investigations of adhesive layers with and without spew fillets. This study determined that the Goland–Reissner model is able to quantitatively capture the critical portions of the stress state in an adhesive layer with spew fillet over the domain where the adherends overlap. Outside this regime (within the spew fillets) the shear and peel stresses decay to zero, making the previous "problems" with the Goland–Reissner approach of little to no consequence.

Vinson [9] performed a preliminary analysis of adhesively bonded asymmetric beams using classical assumptions and the Goland–Reissner adhesive model. This paper suggests that it may be possible to reduce peel stresses by designing the adherends asymmetrically such that the lateral deflection is reduced through bending stretching coupling. This analysis cites a possible competing mechanism related to reduction of bending stiffness resulting from increased asymmetry in laminates of prescribed thickness.

Mortensen and Thomsen [8,10] independently investigated the potential benefits of asymmetric construction in a very comprehensive analysis of adhesively bonded beams. However, their conclusion was that axial stiffness reductions and bending stiffness reductions overwhelm the bending–stretching coupling resulting from asymmetric adherends. This study was limited to prismatic beam adherends and transverse shear deformation was neglected.

Beyond the adhesively bonded structure literature, there have been many investigations into application of asymmetric construction for stress state reductions in composite material plates and shells. A sampling of these studies is [11–14], but there are simply too many of these to cite them encyclopedically. A review of these studies allows the researcher to acquire an intuition about the behavior of composite materials, and to understand what the capabilities of these anisotropic structures are.

This is the context and motivation for this research; the analysis, design and evaluation of adhesively bonded composite material shells and beams. The primary for the present work is to present a solution methodology that can accommodate the geometric and material complications cited previously, and to quantitatively analyze adhesively bonded joints. Using this approach, a series of parametric studies are performed on asymmetrically constructed adhesively bonded joints to evaluate which adherend stiffness parameters have the largest impact on the structural efficiency of these structures.

2. Governing equations

When solving a problem of elasticity for a body, or a system of bodies, one is most often interested in solving the differential equations of motion defined as:

$$\underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{b} = \rho \frac{\partial^2 \underline{u}}{\partial t^2} \tag{1}$$

where $\underline{\nabla}$ is the Del operator, $\underline{\underline{\sigma}}$ is the stress tensor, $\underline{b}$ is the body force vector, ρ is the density of the material, $\underline{u}$ is the displacement vector, and $\frac{\partial^2 \underline{u}}{\partial t^2}$ is the acceleration of the displacement. When one is solving for the equilibrium states of these bodies or systems of bodies, the acceleration goes to zero, thus we have the simpler relationship: