C. R. Mecanique 344 (2016) 151–166

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

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Nonlinear vibrations of buckled plates by an asymptotic numerical method

Lahcen Benchouaf *, El Hassan Boutyour

Department of Applied Physics, Faculty of Sciences and Technology, Hassan 1st University, PO Box 577, Settat, Morocco

A R T I C L E   I N F O

Article history:
Received 27 October 2015
Accepted 5 January 2016
Available online 3 February 2016

Keywords:
Nonlinear vibrations
Buckling
Von Karman plate
Asymptotic numerical method
Harmonic balance method
Finite-element method

A B S T R A C T

This work deals with nonlinear vibrations of a buckled von Karman plate by an asymptotic numerical method and harmonic balance approach. The coupled nonlinear static and dynamic problems are transformed into a sequence of linear ones solved by a finite-element method. The static behavior of the plate is first computed. The fundamental frequency of nonlinear vibrations of the plate, about any equilibrium state, is obtained. To improve the validity range of the power series, Padé approximants are incorporated. A continuation technique is used to get the whole solution. To show the effectiveness of the proposed methodology, numerical tests are presented.

1. Introduction

Vibrations and buckling are common instability phenomena accompanied, generally, by large displacements and important changes in the shape of structures widely used in various industrial fields such as civil engineering, mechanics, aerospace, etc. For appropriate design, it is necessary to develop analytical, numerical or experimental tools able to analyze these problems in order to predict accurately critical loads and natural frequencies (instability and resonance regions). In the literature, various approaches coupling the two problems were developed. A relatively simple one consists, first, in computing static equilibrium branches with the corresponding critical loads and, secondly, in analyzing the vibrations of the structure about a given equilibrium position of the pre- or post-buckled domain. The majority of the realized works, using the indicated approach, concern only linear theory and consider beams, plates or shells structures. These studies show, especially, that the first frequencies can define bifurcation indicators that can be employed advantageously in the non-destructive control. Furthermore, some papers consider non-conservative loads leading to complex instabilities called flutter phenomena. But it is known that, when a shell is deflected more than approximately one-half of its thickness, significant geometrical nonlinearities are induced and a variation of the frequency resonance with the vibration amplitude is shown [1]. However, to the author's knowledge, only a few works have shown an interest in the coupling of the buckling and vibrations and taken into account these nonlinearities [2–12]. Min and Eisley [2] and Tseng and Dugundji [3] adopted analytical procedures based on Galerkin method and modal approximation to study beams subjected to in-plane load. Note that, experimental results were presented in the last paper. Using the same procedures with elliptic integrals, Lesatri and Hanagul [4] studied beams with elastic end restraints. Employing Kirchhoff plate theory and Harmonic balance method, Mahdavi et al. [5] examined the effect of in-plane load on embedded single layer graphene sheet (SLGS) in a polymer

* Corresponding author.
E-mail address: l.benchouaf@uhp.ac.ma (L. Benchouaf).

http://dx.doi.org/10.1016/j.crme.2016.01.002

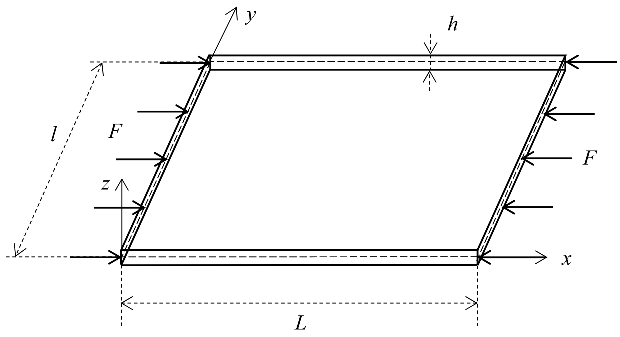

Fig. 1. Geometry and coordinate system of a rectangular plate.

matrix aroused by nonlinear Van Der Waals forces. Shojaei et al. [6] proposed a numerical approach based on Galerkin procedure and a discretization of the space and time domains to investigate Euler–Bernoulli beams with different boundary conditions. Ansari et al. [7] studied microscale Euler–Bernoulli beams employing a couple stress theory. They also considered post-buckled von Karman nanoplates [8]. Based on a high order shear deformation theory, on a Galerkin method and a Newton–Raphson iterative procedure, Girish and Ramachandra [9] investigated laminated composite plates, with initial geometric imperfections and subjected to a uniform temperature distribution through the thickness. Li et al. [10] used von Karman plate theory, Kantorovich time-averaging method and a shooting method to study circular orthotropic plates with a centric rigid mass. Xia and Shen [11,12] considered sandwich plates with functionally graded material FGM face sheets and FGM plates with a layer of piezoelectric actuators in thermal environments and subjected to a compression load. The used formulation is based on a high-order shear deformation theory and takes into account thermo-piezoelectric effects. The motion equations are solved by a perturbation technique.

The aim of the present work consists in studying the nonlinear free vibrations of von Karman plates about a static equilibrium of the pre- or post-buckled domain, by an asymptotic numerical method (ANM) combined to a harmonic balance method. The unknowns of the problem (solution branches, natural frequencies and mode shapes) are determined by a perturbation technique whose terms are computed by a finite-element method. The coupled nonlinear problems (static and dynamic) are transformed into a sequence of linear ones with only two operators to be inverted. In this approach, one searches, first, the equilibrium branches and the bifurcation points. Secondly, the backbone curves, related to the fundamental frequency of nonlinear free vibration of the structure about any equilibrium position of pre and post buckling domain, are determined. At each stage of the proposed algorithm, Padé approximants are incorporated to improve the validity range of the power series and to reduce the computational cost. The whole solution branches at large displacements are derived by the continuation procedure. To show the effectiveness and the reliability of the proposed methodology, numerical tests are presented.

2. Formulation of the problem

The main objective of this paper consists in developing a methodological approach based on the asymptotic numerical method and coupling buckling and nonlinear free vibration of thin plates subjected to uniaxial load. Here, one follows exactly the same methodology as that adopted in [13]. After determining the static fundamental branch, the bifurcation point and the bifurcated branches, the backbone curve corresponding to the fundamental frequency of the nonlinear vibrations of the plate about any equilibrium state of the pre- or post-buckled domain is determined.

2.1. Governing equation of static equilibrium

Let us consider an elastic and homogeneous rectangular plate of thickness h, length L, width l, middle surface Ω, density mass ρ, Young modulus E, Poisson's ratio ν. In a rectangular coordinate reference frame $(O; x, y, z)$, the displacement components of a middle surface point of coordinates (x, y, z) are denoted by u, v, and w in the x, y and z directions, respectively. One assumes that the plate is subjected to a uniform axially compressive force F per unit length, in the x-direction, along the edges $x = 0, L$ (see Fig. 1).

The governing equation of the nonlinear static behavior can be derived by the von Karman theory. To use easily a perturbation technique, a mixed principal is required. The stationarity of the Hellinger–Reissner functional gets [14]:

$$\overline{\mathrm{L}}\big(\mathrm{U}^{\mathrm{s}}\big) + \overline{\mathrm{Q}}\big(\mathrm{U}^{\mathrm{s}}, \mathrm{U}^{\mathrm{s}}\big) - \lambda F = 0 \tag{1}$$

where $\mathrm{U}^{\mathrm{s}} = \{\mathrm{u}^{\mathrm{s}}, \mathrm{v}^{\mathrm{s}}, \mathrm{w}^{\mathrm{s}}, \mathrm{N}^{\mathrm{s}}\}^{\mathrm{t}}$ is a mixed unknown vector, the linear operator $\overline{\mathrm{L}}(\cdot)$ and the quadratic one $\overline{\mathrm{Q}}(\cdot,\cdot)$ are defined by:

$$\big\langle \overline{\mathrm{L}}\big(\mathrm{U}^{\mathrm{s}}\big), \delta\overline{\mathrm{U}}\big\rangle = \int\limits_{\Omega} \big\{\delta\mathrm{N} : \big(\Gamma^{\mathrm{l}}\big(\mathrm{u}^{\mathrm{s}}\big) - [\mathrm{C_m}]^{-1} : \mathrm{N}^{\mathrm{s}}\big) + \delta\Gamma^{\mathrm{l}}(\delta\overline{\mathrm{u}}) : \mathrm{N}^{\mathrm{s}} + \delta\kappa : [\mathrm{C_b}] : \kappa^{\mathrm{s}}\big\} \mathrm{d}\Omega \tag{2}$$