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Transient response of thermoelastic bodies linked by a thin layer of low stiffness and high thermal resistivity

Réponse instationnaire de corps thermoélastiques liées par une couche mince de faible rigidité et haute résistivité thermique

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ABSTRACT

We extend to the thermoelastic case the study [1] devoted to the dynamic response of a structure made of two linearly elastic bodies linked by a thin soft adhesive linearly elastic layer. Once again, a formulation in terms of an evolution equation in a Hilbert space of possible states with finite energy makes it possible to identify the asymptotic behavior, when some geometrical and thermomechanical parameters tend to their natural limits, as the response of two bodies linked by a thermomechanical constraint. The genuine thermomechanical coupling remains in the constraint law only for a specific relative behavior of the parameters.

RÉSUMÉ

On étend au cas thermoélastique l'étude [1] consacrée à la réponse dynamique d'un assemblage de deux corps linéairement élastiques liés par une couche adhésive linéairement élastique mince et molle. À nouveau, une formulation en terme d'équations d'évolution dans un espace de Hilbert d'états possibles d'énergie finie permet d'identifier le comportement asymptotique, lorsque des paramètres géométriques et thermomécaniques tendent vers leurs limites naturelles, comme la réponse de l'assemblage des deux corps par une liaison thermomécanique. Le couplage thermomécanique initial perdure dans la loi de la liaison uniquement pour un comportement relatif particulier des paramètres.

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1. Introduction

Adhesively bonded joints are an attractive way to put together the components of a structure. As in several situations thermal effects are not negligible, we extend a previous study [1] devoted to a linearly elastic material to the linearly thermoelastic case. Taking advantage of the coupling between mechanical and thermal effects, it is still possible to formulate the problem of determining the transient response of a structure made of two linearly thermoelastic bodies perfectly connected by a thin soft thermoelastic layer with high thermal resistivity in terms of an evolution equation in a Hilbert space of possible states (displacement, temperature, velocity) with finite energy. Hence it is possible to adopt the strategy of [1,2] in order to, first, obtain existence and uniqueness results and, then, to study the asymptotic behavior when some geometrical and thermomechanical data, now regarded as parameters, tend to their natural limits. The limit behavior which supports our proposal of a simplified but accurate enough model for the initial physical situation, corresponds to the dynamic response to the initial load of two linearly thermoelastic bodies connected by a thermomechanical constraint along the surface the adhesive layer shrinks to. The structure of the constitutive equations of the constraint is similar to the one of the layer with coefficients depending on the relative behaviors of the parameters but the thermomechanical coupling is maintained only for a particular relative behavior.

2. Setting the problem

We consider a structure consisting of two thermoelastic bodies (adherents) bonded by a thin thermoelastic layer (adhesive). The entire system occupies the domain $\Omega \subset \mathbb{R}^3$ with a Lipschitz-continuous boundary $\partial\Omega$. Let (Γ_0^M, Γ_1^M) and (Γ_0^T, Γ_1^T) be two partitions of $\partial\Omega$ with $\mathcal{H}_2(\Gamma_0^T) > 0$ and $\mathcal{H}_2(\Gamma_0^M) > 0$, where $\mathcal{H}_2$ is the two dimensional Hausdorff measure. We denote the orthonormal canonical basis of $\mathbb{R}^3$, assimilated to the physical Euclidean space, by $\{e_1, e_2, e_3\}$ and for all (x_1, x_2, x_3) in $\mathbb{R}^3$, $\hat{x}$ stands for (x_1, x_2). The intersection S of Ω with $\{x_3 = 0\}$ is supposed to have a positive Hausdorff measure and it is also assumed that there exists $\varepsilon_0 > 0$ such that $B_{\varepsilon_0} = \{(\hat{x}, x_3) \in \Omega;\ |x_3| < \varepsilon_0\}$ is equal to $S \times (-\varepsilon_0, \varepsilon_0)$. Let $\varepsilon < \varepsilon_0$, then the adhesive occupies the layer B_ε, while each of the two adherents occupies $\Omega_\varepsilon^\pm := \{x \in \Omega;\ \pm x_3 > \varepsilon\}$ and let $\Omega_\varepsilon = \Omega_\varepsilon^+ \cup \Omega_\varepsilon^-$. Adherents and adhesive are assumed to be perfectly stuck together along $S_\varepsilon = S_\varepsilon^+ \cup S_\varepsilon^-$, $S_\varepsilon^\pm := \{x \in \Omega;\ x_3 = \pm\varepsilon\}$. The structure is clamped on Γ_0^M, maintained at a uniform temperature T_0 on Γ_0^T, subjected to body forces of density f, to surface forces of density g_M on Γ_1^M and to thermal flux g^T on Γ_1^T.

The whole structure is modeled as linearly thermoelastic in the following way. Let $(\rho, \beta, \alpha, \kappa, a) \in L^\infty(\Omega; \mathbb{R} \times \mathbb{R} \times S^3 \times S^3 \times \mathrm{Lin}(S^3))$ satisfying

$$\begin{cases} \exists(\rho_m, \beta_m, \kappa_m, a_m) \in (0, +\infty)^4 \\ \rho(x) \ge \rho_m, \quad \beta(x) \ge \beta_m, \quad \alpha(x) \ge 0, \quad \kappa(x)\xi \cdot \xi \ge \kappa_m |\xi|^2, \quad \forall \xi \in \mathbb{R}^3, \\ \quad a(x)e \cdot e \ge a_m |e|^2, \quad \forall e \in S^3, \quad \text{a.e. } x \in \Omega \end{cases} \tag{1}$$

where S^3 is the space of 3×3 symmetric matrices with the usual inner product and norm denoted by $\cdot$ and $|\cdot|$, as in $\mathbb{R}^3$, and $\mathrm{Lin}(S^3)$ is the space of linear mapping from S^3 to S^3. The mass density, the specific heat coefficient, the thermal dilatation, the heat conductivity and the elasticity coefficients in the adherents are ρ, β, α, κ and a, respectively, while the positive numbers $\widetilde{\rho}$, $\widetilde{\beta}$, $\widetilde{\alpha}$, $\widetilde{\kappa}$, λ and μ denote the mass density, the specific heat coefficient, the thermal dilatation, the heat conductivity and the Lamé coefficients in the adhesive assumed to be isotropic and homogeneous. Thus problem $(\mathcal{P}_h)$ of determining the evolution of the assembly involves the quintuplet $h = (\varepsilon, \lambda, \mu, \kappa, \gamma)$ of data where $\gamma = (3\lambda + 2\mu)\widetilde{\alpha}$ and thereafter all the fields will be indexed by h. In the following, the upper dot denotes the differentiation with respect to time t, $e(u)$ is the linearized strain tensor associated with the displacement field u. Hence problem $(\mathcal{P}_h)$ reads as:

$$(\mathcal{P}_h) \quad \begin{cases} \rho \ddot{u}_h = \operatorname{div} \sigma_h + f, \qquad T_0 \beta \dot{\theta}_h = \operatorname{div} q_h - T_0 a\alpha \cdot e(\dot{u}_h) \quad \text{in } \Omega_\varepsilon \\ \sigma_h = a\big(e(u_h) - \theta_h \alpha\big), \qquad q_h = \kappa \nabla \theta_h \quad \text{in } \Omega_\varepsilon \\ \widetilde{\rho} \ddot{u}_h = \operatorname{div} \sigma_h + f, \qquad T_0 \widetilde{\beta} \dot{\theta}_h = \operatorname{div} q_h - T_0 \gamma \operatorname{tr} e(\dot{u}_h) \quad \text{in } B_\varepsilon \\ \sigma_h = \lambda \operatorname{tr} e(u_h) Id + 2\mu e(u_h) - \gamma \theta_h Id, \qquad q_h = \widetilde{\kappa} \nabla \theta_h \quad \text{in } B_\varepsilon \\ \sigma_h \nu = g^M \quad \text{on } \Gamma_1^M, \qquad q_h \cdot \nu = g^T \quad \text{on } \Gamma_1^T, \qquad u_h = 0 \quad \text{on } \Gamma_0^M, \qquad \theta_h = 0 \quad \text{on } \Gamma_0^T \\ u_h(x, 0) = u_h^0(x), \qquad \dot{u}_h(x, 0) = v_h^0(x), \qquad \theta_h(x, 0) = \theta_h^0(x), \qquad \text{a.e. } x \in \Omega \end{cases} \tag{2}$$

where u_h, θ_h, σ_h and q_h are the fields of displacement, temperature increment with respect to T_0, the stress tensor and the heat flux vector, respectively, while u_h^0, $v_h^0(x)$, θ_h^0 are the initial conditions. The symbols Id and ν refer to the 3×3 identity matrix and the outward unitary normal to $\partial\Omega$.

3. Existence and uniqueness result for $(\mathcal{P}_h)$

Assuming that

$$(H_1): \quad \big(f, g^M, g^T\big) \in C^{0,1}\big([0, T]; L^2\big(\Omega; \mathbb{R}^3\big)\big) \times C^{2,1}\big([0, T]; L^2\big(\Gamma_1^M; \mathbb{R}^3\big)\big) \times C^{2,1}\big([0, T]; L^2\big(\Gamma_1^T\big)\big)$$