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Refined theory of bi-layer beams for a transversely isotropic body

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ABSTRACT

Based on elastic theory, the refined theory of bi-layer beams for a transversely isotropic body is studied. Using the Elliott–Lodge (E–L) solution and Luré method, the refined theory of beams is derived from continuity conditions without ad hoc assumptions. It is shown that the displacements and stresses of the beam can be represented by displacements and stresses of the interface of two layers of different materials. The governing equations about the transverse displacement of the interface can be obtained directly from the refined theory under transverse surface loading. Approximate solutions are derived for beams by dropping terms of high order. In addition, one example is examined to illustrate the application of the theory proposed in this paper.

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1. Introduction

With the development of engineering, monolayer structures cannot meet demand for high-strength, high-modulus and low-density industrial materials, and the laminated composite structures have been used in more and more fields. The bi-layer beams as a special laminated composite structure attracts broad attention. In general, two different models have been used to study bi-layer beams: single-layer theory and layerwise theory [1]. Using single-layer theory, the whole plate is considered as one monolayer structure, and a magnified error will appear. Using layerwise theory, each layer is analyzed, and the precision of layerwise theory will be affected by monolayer structure theory. Moreover, Carrera [2,3] makes a summing up of the theories for multilayered structures and gives one-dimensional formulations for the analysis of multilayered structures.

Research ideas of refined analysis were proposed by Cheng [4] in 1979. A parallel development of Cheng's theory has been obtained by Barrett and Ellis [5] for isotropic plates under transverse surface loadings. Wang and Shi [6] further study Cheng's refined theory by using the Papkovitch–Neuber solution and discussed isotropic plate in the case of surface transverse load. They finally got the plate deflection control equations and shear control equations. Luo and Wang [7] obtained the refined theory of generalized plane-stress problems in elasticity.

Wang [8] applied Cheng's refined theory approach to the study of transversely isotropic plate, and a refined equation for transversely isotropic plates with homogeneous boundary conditions was obtained. Wang substituted the sum of the general integrals of the three differential equations of the plate problem for the solution to the refined equation, but he did not prove the rationality of the substitution. Zhao et al. [9] studied the transcendental equation of the transversely isotropic plate, and proved the rationality of the substitution.

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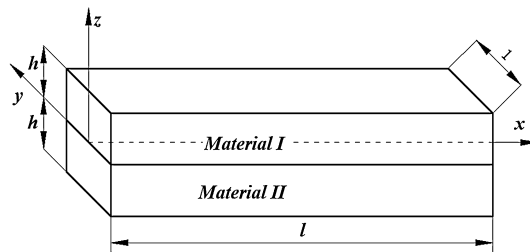


Fig. 1. Schematic diagram of the bi-layer beam structure.

Cheng’s refined theory was extended to transversely isotropic plates under transverse surface loadings by Yin and Wang [10] through an Elliott–Lodge (E–L) solution. Gao and Wang [11,12] extended Cheng’s theory to monolayer isotropic beams, and gave the refined theory for rectangular elastic beam problems. Gao et al. [13] got the refined theory of beams for a transversely isotropic body. Lu et al. [14] got the refined theory of transversely isotropic thermoporoelastic beam. Based on a refined theory of a single layer, Zhao and others [15,16] got the refined analysis of bi-layer beams and a refined theory of sandwich beams without an ad hoc assumption.

It is the purpose of this paper to extend our previous work to the bi-layer beams for a transversely isotropic body and the refined theory is given.

2. Structure and general solutions

In this paper, we study the bi-layer beams. Two beams are made of different transversely isotropic materials. The x – y plane of each layer beam is an isotropic plane and the interface of two materials is also an isotropic plane, and $z = 0$. As shown in Fig. 1, the isotropic planes (x – y plane, $z = 0$) of the two beams are stacked. The beam length in the x -direction is l , the beam width in the y -direction is 1, the beam height in the z -direction is $2h$, and $l \gg 2h \gg 1$. Therefore, it is plausible to set the components of stress $\sigma_y = \tau_{xy} = \tau_{yz} = 0$. This is a plane stress assumption, and the width in the y -direction is stress free. The mark of superscript (1) and (2) denote the top beam and the bottom beam of the bi-layer beam, respectively. And we set the nonhomogeneous boundary conditions to be:

$$\sigma_{zx}^{(1)} = 0, \quad \sigma_{zz}^{(1)} = \frac{q}{2} \quad (\text{on } z = h); \quad \sigma_{zx}^{(2)} = 0, \quad \sigma_{zz}^{(2)} = -\frac{q}{2} \quad (\text{on } z = -h).$$

The constitutive equations for the transversely isotropic body in two-dimensional linear elasticity are described to be:

$$\begin{cases} \sigma_{xx}^{(i)} = c_{11}^{(i)} \frac{\partial u_x^{(i)}}{\partial x} + c_{13}^{(i)} \frac{\partial u_z^{(i)}}{\partial z} \\ \sigma_{zz}^{(i)} = c_{13}^{(i)} \frac{\partial u_x^{(i)}}{\partial x} + c_{33}^{(i)} \frac{\partial u_z^{(i)}}{\partial z} \\ \sigma_{xz}^{(i)} = c_{44}^{(i)} \left(\frac{\partial u_z^{(i)}}{\partial x} + \frac{\partial u_x^{(i)}}{\partial z} \right) \end{cases} \quad (i = 1, 2) \tag{1}$$

The general solution to the transversely isotropic elastic body is:

$$u_x^{(i)} = (\psi_1^{(i)} + \psi_2^{(i)})_x, \quad u_z^{(i)} = (k_1^{(i)} \psi_1^{(i)} + k_2^{(i)} \psi_2^{(i)})_{,z} \tag{2}$$

where the constants $k_1^{(i)}$ and $k_2^{(i)}$ satisfy:

$$\begin{aligned} \frac{c_{11}^{(i)}}{c_{44}^{(i)} + (c_{13}^{(i)} + c_{44}^{(i)})k_1^{(i)}} &= \frac{c_{13}^{(i)} + c_{44}^{(i)}(1 + k_1^{(i)})}{c_{33}^{(i)}k_1^{(i)}} = (s_1^{(i)})^2 \\ \frac{c_{11}^{(i)}}{c_{44}^{(i)} + (c_{13}^{(i)} + c_{44}^{(i)})k_2^{(i)}} &= \frac{c_{13}^{(i)} + c_{44}^{(i)}(1 + k_2^{(i)})}{c_{33}^{(i)}k_2^{(i)}} = (s_2^{(i)})^2 \end{aligned} \quad (i = 1, 2) \tag{3}$$

and $(s_1^{(i)})^2$ and $(s_2^{(i)})^2$ are two characteristic roots of the following quadratic algebra equation of $(s^{(i)})^2$,

$$c_{33}^{(i)}c_{44}^{(i)}(s^{(i)})^4 + [(c_{13}^{(i)})^2 + 2c_{13}^{(i)}c_{44}^{(i)} - c_{11}^{(i)}c_{33}^{(i)}](s^{(i)})^2 + c_{11}^{(i)}c_{44}^{(i)} = 0 \tag{4}$$

We obtain the two roots $(s_1^{(i)})^2$ and $(s_2^{(i)})^2$ of the algebra equation (4) and assume that they are distinct. So the potential functions $\psi_i^{(1)}$ and $\psi_i^{(2)}$ satisfy the following equation:

$$\nabla_i^2 \psi_i^{(1)} = 0, \quad \nabla_i^2 \psi_i^{(2)} = 0 \quad (i = 1, 2) \tag{5}$$

with $\nabla_i^2 = \partial_x^2 + \partial_z^2 / (s_i^{(1)})^2$, $\nabla_i^2 = \partial_x^2 + \partial_z^2 / (s_i^{(2)})^2$, $\partial_x = \partial / \partial x$, and $\partial_z = \partial / \partial z$.

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