C. R. Mecanique 343 (2015) 635–644

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

www.sciencedirect.com

Acoustic metamaterials and phononic crystals

Sound absorption by subwavelength membrane structures: A geometric perspective

Min Yang [a], Yong Li [a], Chong Meng [a], Caixing Fu [a], Jun Mei [b], Zhiyu Yang [a], Ping Sheng [a,*]

[a] *Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China*
[b] *Department of Physics, South China University of Technology, Guangzhou 510640, China*

A R T I C L E I N F O

Article history:
Received 25 June 2015
Accepted 25 June 2015
Available online 21 August 2015

Keywords:
Metamaterials
Decorated membrane resonator
Low-frequency sounds absorption
Conservation of mean complex pressure amplitudes
Thin film absorption limit
Hybrid resonances
Metasurface
Subwavelength total absorption

A B S T R A C T

Decorated membranes comprising a thin layer of elastic film with small rigid platelets fixed on top have been found to be efficient absorbers of low-frequency sound. In this work we consider the problem of sound absorption from a perspective aimed at deriving upper bounds under different scenarios, i.e., whether the sound is incident from one side only or from both sides, and whether there is a reflecting surface on the back side of the membrane. By considering the negligible thickness of the membrane, usually on the order of a fraction of one millimeter, we derive a relation showing that the sum of the incoming sound waves' (complex) pressure amplitudes, averaged over the area of the membrane, must be equal to that of the outgoing waves. By using this relation, and without going to any details of the wave solutions, it is shown that the maximum absorption achievable from one-sided incidence is 50%, while the maximum absorption with a back-reflecting surface can reach 100%. The latter was attained by the hybridized resonances. All the results are shown to be in excellent agreement with the experiments. This generalized perspective, when used together with the Green function's formalism, can be useful in gaining insights into the constraints on what are achievable in scatterings and absorption by thin film structures and delineating them.

1. Introduction

A sound absorber converts the airborne acoustic energy into thermal motions via irreversible processes. High efficiency of such processes requires not only properly matched impedance for the absorber, but also the dissipative capability to absorb the incident energy. Traditional means of acoustic absorption make use of porous and fibrous materials [1], gradient index materials, or perforated panels [2] with tuned cavity depth behind the panels. They generally result in either imperfect impedance matching to the incoming wave, or very bulky structures with dimensions comparable to the wavelength, usually on the order of meters for low-frequency airborne sound.

Membrane-type acoustic metamaterials, consisting of decorated membrane resonators (DMR) of various forms, have been shown to display diverse functionalities, such as efficient reflection [3–7], enhanced transmission [8], reversed Doppler effect [9], and near-field amplification [10]. The reason for such extraordinary behaviors can be attributed to the effective

* Corresponding author.
E-mail address: sheng@ust.hk (P. Sheng).

http://dx.doi.org/10.1016/j.crme.2015.06.008

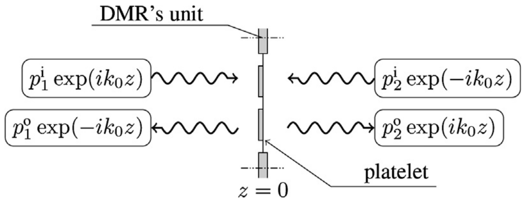

Fig. 1. Schematic illustration of the scattering process from a DMR unit. With left-incident sound wave's pressure denoted as $p_1^{\rm i}\exp({\rm i}k_0z)$ and that from the right as $p_2^{\rm i}\exp(-{\rm i}k_0z)$, scattering generates two outgoing waves $p_1^{\rm o}\exp(-{\rm i}k_0z)$ and $p_2^{\rm o}\exp({\rm i}k_0z)$. Here k_0 is the sound wave's wavevector in the air, and the double dot–dash lines stand for periodic boundary conditions. According to the conservation of the mean complex pressure amplitude (MCPA), $p_1^{\rm i}+p_2^{\rm i}=p_1^{\rm o}+p_2^{\rm o}$.

negative mass density [3,11] and/or negative refractive index [12,13] induced by the DMR's subwavelength resonances. Recent works also show that the DMR can efficiently absorb low-frequency sound even with its negligible thickness [14,15], a capability clearly beyond what is achievable by conventional acoustic absorbers. This has been attributed to the high energy density of DMR's lateral resonances. Hybridization of different resonances can even lead to perfect absorptions in the deep-subwavelength regime, thereby realizing the exact time-reversed counterpart of an acoustic point source [16]. The latter is known to have important applications for time-reversal wave technology [17]. By using hybridized modes, an acoustic metasurface can be realized with an array of such acoustic "sinks" with extraordinary sound absorption characteristics [18].

In this work, we present a generalized perspective, based on DMR's special geometric characteristic, for understanding DMR's sound absorption behaviors. It is shown that, owing to the membrane's negligible thickness, there is an equality relating the sum of mean complex pressure amplitudes (MCPA) of the incoming waves, averaged over the area of the DMR, to that of the outgoing waves. By drawing analogy to momentum conservation law involving two equal-mass particles, it is easily seen that there is a component of the total incident energy, corresponding to the center of mass motion in the two particles collision, which is always conserved and therefore cannot be dissipated. Hence only the energy in excess of the conserved component is available for dissipation. This observation leads to upper limits for DMR's absorption performance under various scenarios. In particular, for a wave incident from one side only, only half of the incoming energy is available for dissipation, thus the absorption percentage cannot exceed 50% [15]. We verify this conclusion by experimentally observing sound scattering from DMRs, and the results show excellent agreement with theoretical predictions. Such general considerations also show that absorption higher than 50% can be achieved either by allowing waves to be incident from both sides of the membrane, such as in the coherent perfect absorption (CPA) scenario [19–21], or if multiple scatterings are introduced into the system by introducing a back-reflecting surface. As an example, the acoustic metasurface with hybrid resonances, which has been demonstrated to completely absorb waves that are incident from one side, is analyzed in light of the MCPA conservation. The outcome, while matching the results previously reported [18], also yields additional insights.

In what follows, Section 2 shows how the area-averaged response of a subwavelength membrane structure can be reduced to a one-dimensional problem, and how the related physics, as embedded in the Green function formalism, is considerably simplified as a result. In Section 3, we derive a general conservation rule governing the scatterings from thin membrane structures, and show how some conclusions can be easily derived from this rule without having to resort to the wave equation. In Section 4 we analyze the problem of an efficient thin membrane absorber using this generalized perspective, and verify experimentally the derived conservation rule. In Section 5, the problem of a membrane resonator coupled to a thin, sealed air cell is analyzed, leading to the condition for attaining perfect absorption through hybrid resonances. This is followed by a brief summary in Section 6, which concludes the article. In Appendix A, we derive the formulas for obtaining the theoretically relevant parameters from four-probe impedance tube data.

2. Surface impedance and dissipation

A DMR unit comprises a uniformly stretched elastic membrane decorated with relatively rigid platelets. It is important for our subsequent considerations to note that the thickness of the membrane is negligible (∼200 μm), while the cross sectional dimension of the membrane is subwavelength in scale for the frequency regime of interest. Without loss of generality, we can consider it as an inhomogeneous membrane that can vibrate and scatter the incoming plane waves, as shown in Fig. 1.

Central to the understanding of DMR's acoustic behaviors is that only the piston-like component of its averaged displacement, $\langle W\rangle$, couples to the radiative wave modes. Here W denotes the normal displacement field of the membrane, and the angular bracket denotes surface averaging. However, the variance of the displacement, $\delta W \equiv W-\langle W\rangle$, is decoupled from the radiation modes and can be characterized as "deaf." The reason of this decoupling can be seen from the Fourier wavevectors $\mathbf{k}_\parallel$ that delineate the lateral spatial pattern of W. For the δW component, the relevant $\mathbf{k}_\parallel$ components must have magnitudes that satisfy the inequality $|\mathbf{k}_\parallel|>2\pi/\lambda$, where λ is the sound wavelength in air, owing to the membrane's subwavelength cross sectional dimension. From the displacement continuity condition and the wave dispersion relation, we have $(k_\parallel)^2+(k_\perp)^2=(2\pi/\lambda)^2$ for the acoustic wave in the air, where $k_\perp$ denotes the wavevector component normal to the membrane. It follows that the δW component couples only to the evanescent waves, as its associated $k_\perp$ must be imaginary. In contrast, because the $\mathbf{k}_\parallel$ components for $\langle W\rangle$ have a distribution that peaks at $\mathbf{k}_\parallel=0$, it can couple to the