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Acoustic metamaterials and phononic crystals

Torsional topology and fermion-like behavior of elastic waves in phononic structures

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A one-dimensional block-spring model that supports rotational waves is analyzed within Dirac formalism. We show that the wave functions possess a spinor and a spatio-temporal part. The spinor part leads to a non-conventional torsional topology of the wave function. In the long-wavelength limit, field theoretical methods are used to demonstrate that rotational phonons can exhibit fermion-like behavior. Subsequently, we illustrate how information can be encoded in the spinor-part of the wave function by controlling the phonon wave phase.

1. Introduction

Our understanding of elastic waves has been nourished essentially by the simple paradigm of the plane wave and its periodic counterpart (the Bloch wave) in periodic media. Significant progress has been achieved in unraveling the behavior of elastic waves in phononic crystals and acoustic metamaterials [1]. However, most of this progress has been achieved through exploration of the geometrical complexity of these media. Inspired by the discovery of topological insulators for which the electronic wave function is supported in momentum space by manifolds with non-conventional topologies [2], recent studies have shown the possibility of achieving electromagnetic waves [3,4] and acoustic waves with non-conventional topologies [5,6]. In the present paper, we investigate the properties of wave functions in phononic structures supporting rotational waves which can exhibit wave functions that take the form of spinors [7]. The non-conventional torsional topology of the wave function of elastic waves with spinor characteristics leads to a constraint on the wave function reminiscent of the Pauli-exclusion principle. This type of constraint introduces the notion of fermion-like behavior of elastic waves. Topological constraints on elastic waves promises to offer unique, robust designs and new device functionalities to phononic systems by providing immunity to performance degradation caused by imperfections [8,9] or information coding and processing in the phase of the waves.

In reference [7], we reported on an investigation of a 2D PC constituted of stiff polymer inclusions in a soft elastomer matrix. The 2D PC composed of a square array of polystyrene (PS) inclusions in a polydimethylsiloxane (PDMS) elastomer matrix was shown to support rotational waves. Of particular interest were modes where the PS inclusions and the region of the matrix separated by the inclusions rotate out of phase but also in phase. Following Peng et al. [10], who demonstrated that a 1D lumped mass model could be used to describe rotational modes in a 2D PC, we introduced a 1D mass spring phononic structure that could also support rotational waves. The 1D model was used to reproduce the dispersion relations of the 2D system in a certain range. We then showed within Dirac's formalism that the wave function for rotational waves possessed a spinor-part and a spatio-temporal part.

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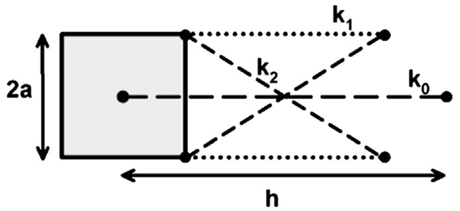

Fig. 1. Schematic illustration of a discrete micromechanics model that supports rotational waves. Unit cell of a monoblock model with elements (blocks) connected by three types of harmonic springs (spring constants k_0, k_1, and k_2). Each element possesses translational longitudinal (u), shear (v) and rotational (φ) degrees of freedom.

In the present paper, we investigate the geometric topology of the spinor-part of the wave function. We show that the spinor-part imparts a non-conventional topology to the wave function. In particular, the spinor part of the elastic wave function is supported in momentum space by a manifold with torsional topology. Furthermore, in the long-wavelength limit of the 1D model, we integrate the concepts of phononic structures that support rotational waves with that of a Dirac representation of elastic waves within a field theoretical framework. This leads to the observation of fermion-like behavior resulting from the spinor part of the wave function. In particular within the context of a "second quantization" of the rotational waves, we show that the properties of their associated spinor lead to anticommutation rules for the creation and annihilation operators.

After showing the possibility of fermion-like constraints on the elastic waves, a second question related to the more practical aspect of this behavior is also posed: what are the implications of fermion-like phonon behavior? A partial answer to that question is offered by introducing the concept of phase control of phonons through the specificity of their fermion character. To that effect, we introduce the concept of a phononic structure-based φ(phase)-bit. We demonstrate that one can operate on the φ-bit through the structure's physical parameters to transform the spinor state of the wave function. More specifically, one can operate on and measure the spinor state of phonons replicating known operations that are more commonly executed on fermions states like spins.

This paper is organized as follows: in Section 2, we describe the 1D discrete mass-spring model that supports rotational modes. This model forms the foundation of the development of a field theoretical representation of rotational modes in phononic structures. The wave equation associated with that model is shown to be isomorphic to the Klein–Gordon equation. That equation is therefore factored using Dirac formalism. We obtain the wave functions and investigate their non-conventional topology in momentum (k) space. In Section 3, we use quantum field theoretic methodologies to analyze the properties of the wave functions in the long-wavelength limit. In particular we find constraints on the wave function reminiscent of fermion-like behavior. An application of these findings to the encoding and processing of information in the wave function is introduced in Section 4. Finally conclusions are drawn in Section 5.

2. Nonconventional topology of rotational waves in phononic crystals

2.1. One-dimensional discrete micromechanics model

Here, we describe a 1D mass-spring model that was shown in reference [7] to reproduce the dispersion relations of a 2D phonon crystal that supports rotational waves. This model is based on a discrete linear one-dimensional micromechanics model that includes longitudinal, shear and rotational degrees of freedom [11,12]. This 1D discrete lattice model consists of an infinite chain of square block elements connected with multiple harmonic springs. Each element in the model is considered to have two translational degrees of freedom (displacement in the x and y directions) and one rotational degree of freedom (rotation about an axis perpendicular to the xy-plane). Fig. 1 shows the repeatable unit cells for a monoblock lattice models with periodicity (h).

Three different harmonic springs (spring constants k_0, k_1, and k_2) connect different parts of the block elements. The element in Fig. 1 has mass (m) and moment of inertia (I). The block constituting the nth unit cell has x-displacement (u_n), y-displacement (v_n) and rotation component (φ_n). u_n and v_n represent displacements associated with longitudinal and transverse vibrations, respectively. The potential energy associated with the elastic connections of elements (n) and ($n+1$) in the monoblock chain is written as follows:

$$E_{n,n+1} = \frac{1}{2}K_0(u_{n+1}-u_n)^2 + \frac{1}{2}K_1\left[(v_{n+1}-v_n) + \frac{h}{2}(\varphi_{n+1}+\varphi_n)\right]^2 + \frac{1}{2}K_2(\varphi_{n+1}-\varphi_n)^2 \tag{1}$$

where $K_0 = (\frac{k_0}{h^2} + \frac{2k_1}{l^2} + \frac{2k_2 l^2}{l_d^4})$, $K_1 = (\frac{2k_2(2a)^2}{l_d^4})$, $K_2 = (\frac{2a^2 k_1}{l^2})$, $l = h-(2a)$, $l_d = \sqrt{(l^2+(2a)^2)}$. Accordingly, the equations of motion for the element in the nth unit cell of the monoblock lattice are written as:

$$m\frac{d^2u_n}{dt^2} = K_0(u_{n+1}-2u_n+u_{n-1}) \tag{2}$$