C. R. Mecanique 343 (2015) 559–570

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

www.sciencedirect.com

Discrete simulation of fluid dynamics

Simulation of residual oil displacement in a sinusoidal channel with the lattice Boltzmann method

Hiroshi Otomo [a,*], Hongli Fan [a], Randy Hazlett [b], Yong Li [a], Ilya Staroselsky [a], Raoyang Zhang [a], Hudong Chen [a]

[a] Exa Corporation, 55 Network Drive, Burlington, MA 01803, USA
[b] The University of Tulsa, 800 South Tucker Drive, Tulsa, OK 74104, USA

ARTICLE INFO

Article history:
Received 24 December 2014
Accepted 17 April 2015
Available online 12 August 2015

Keywords:
Multi-component
LBM
Critical pressure
Sinusoidal channel

ABSTRACT

We simulate oil slug displacement in a sinusoidal channel in order to validate computational models and algorithms for multi-component flow. This case fits in the gap between fully realistic cases characterized by complicated geometry and academic cases with simplistic geometry. Our computational model is based on the lattice Boltzmann method and allows for variation of physical parameters such as wettability and viscosity. The effect of variation of model parameters is analyzed, in particular via comparison with analytical solutions. We discuss the requirements for accurate solution of the oil slug displacement problem.

1. Introduction

Transport properties of immiscible fluids in porous media have been extensively investigated because of their practical and fundamental importance. The challenges here are due to complex multi-component and multi-scale physics, as well as complex geometry.

One practically important example of such flows is in petroleum reservoir engineering, where the minimum pressure required for removing residual oil is one of the critical rock properties. Due in large part to the limitations of physical experiment, engineers and scientists are increasingly considering numerical simulation, which is very difficult as well. In addition to the usual challenges of complex flow modeling, pores and voids in the rocks have nontrivial topological and geometrical structure. Moreover, the wettability of pore walls that strongly influences the flow through the rock, is hard to take into account in the computational model. In a real rock, the wettability is variable and depends on such factors as mineral composition, microscopic surface roughness, pore shapes, and the adsorption effects [1].

One of the promising computational approaches for modeling this class of flows is the lattice Boltzmann method (LBM). Its first advantage is its framework that is based on the mesoscopic kinetic theory. Compared with Navier–Stokes-based formalisms, it describes small scale effects more naturally. For example, interfaces between different components are automatically determined once the species' interactions are defined. Detailed modeling of the wall boundaries is also more natural [2]. Second, the LBM involves only cubic volume lattices that do not adapt to solid boundaries, so that the volume meshing can be made simple and automatic [3–6]. Third, the LBM generally has highly parallel computational performance since most of operations are performed locally.

* Corresponding author.
E-mail address: hotomo@exa.com (H. Otomo).

http://dx.doi.org/10.1016/j.crme.2015.04.005
1631-0721/

A number of previous studies that use the LBM for simulation of rock samples and porous media show promising results [7–13]. However, when a realistic case is simulated it may be difficult to identify the model features that are responsible for deviation of computational results from the experiment and theory. Therefore, it is desirable to have more basic cases that fit in the gap between fully featured realistic cases and simple academic benchmark tests. Most of such previously studied simple cases include the capillary rising, the Hagen–Poiseuille flow, the Couette flow, and droplet in free space under specific conditions [2,14–16]. As stated in these papers, essential issues relevant to realistic cases are not accounted for, such as the resolution dependence for complex geometry, transition from steady to unsteady flow regimes, the hysteresis effects, etc.

In this work, we focus on computation of the minimum pressure required for removing residual oil, which is called the critical pressure, in a sinusoidal channel using a multi-component LBM approach. This geometry can be viewed as a simple prototype of porous media [13]. The existence of analytical solution for the critical pressure in this case [17–19] makes it possible to evaluate the accuracy of predicting the transition from the static to moving slug. Furthermore, the effects due to resolution, viscosity, and wettability variation upon the quality of numerical results can be evaluated using this prototype model of porous media.

This paper is organized as follows. In Section 2, we review the LBM formalism for multi-component flow. In Section 3, we report simulation results. The first case is a two-dimensional droplet in free space, which serves to determine the surface tension. The second case is the two component Hagen–Poiseuille flow, that is used to test the viscous effect. The third case is a two-dimensional slug between flat plates, that is used to define the relation between the contact angle and the corresponding control parameter of the model. After the model parameters are chosen based on these results, the critical pressure for an oil slug in a sinusoidal channel is investigated. In Section 4, we summarize the main findings and discuss some potential extensions of this study.

2. The lattice Boltzmann method for multi-component flow

Since more than twenty years ago, the LBM has been developed in various ways for simulation of immiscible fluid flows [20]. The LBM model we developed and applied in this study is originated from the well-known Shan–Chen model [21,22]. Combined with other recent LBM advancements [23–27], our model provides accurate and stable results, in particular for small viscosity and in arbitrary geometry. The formalism that we use is briefly described below.

The general lattice Boltzmann (LB) equation for multi-component fluid flow, for example that consisting of oil and water, is as follows:

$$f_i^\alpha(\boldsymbol{x}+\boldsymbol{c}_i\Delta t, t+\Delta t) - f_i^\alpha(\boldsymbol{x}, t) = \mathcal{C}_i{}^\alpha + \mathcal{F}_i^\alpha \tag{1}$$

where f_i^α is the density distribution function of each fluid component, $\boldsymbol{c}_i$ is the discrete particle velocity and α is an index for the oil or water component, $\alpha = \{o, w\}$. The D3Q19 [28] lattice model is adopted here so that the i ranges from 1 to 19. The collision term $\mathcal{C}_i{}^\alpha$ defines relaxation of particles' distribution functions towards their equilibrium states. $\mathcal{F}_i^\alpha$ is the term associated with the inter-component interaction force. The most popular and simple form of the collision operator is the BGK operator [28–31] with a single relaxation time,

$$\mathcal{C}_i{}^\alpha = -\frac{1}{\tau^\alpha}(f_i^\alpha - f_i^{\mathrm{eq},\alpha}) \tag{2}$$

After rearrangement of some terms, the two above equations can be written in the following form,

$$f_i^\alpha(\boldsymbol{x}+\boldsymbol{c}_i\Delta t, t+\Delta t) = f_i^{\mathrm{eq},\alpha}\left(\rho^\alpha, \boldsymbol{u}\right) + \left(1 - \frac{1}{\tau_{\mathrm{mix}}}\right) f_i'^\alpha + \mathcal{F}_i^\alpha \tag{3}$$

Here τ_{mix} is the "mixed" relaxation time that is related to the kinematic viscosity of the mixture of components:

$$\tau_{\mathrm{mix}} = (\nu_{\mathrm{mix}}/T_0) + \frac{1}{2} \tag{4}$$

$$\nu_{\mathrm{mix}} = \left(\rho^{\mathrm{o}}\nu^{\mathrm{o}} + \rho^{\mathrm{w}}\nu^{\mathrm{w}}\right)/\left(\rho^{\mathrm{o}} + \rho^{\mathrm{w}}\right) \tag{5}$$

where $T_0 = 1/3$ is the lattice temperature in D3Q19. The function $f_i'^\alpha$ is the nonequilibrium particle distribution for each fluid component. It is important that instead of using the standard BGK form $f_i'^\alpha = f_i^\alpha - f_i^{\mathrm{eq},\alpha}$, a regularized collision procedure is applied in this work in order to calculate $f_i'^\alpha$,

$$f_i'^\alpha = \Phi^\alpha : \Pi^\alpha \tag{6}$$

Here Φ is a regularization operator that uses Hermite polynomials and Π^α is the nonequilibrium part of the momentum flux. The basic concept of regularized collision procedure can be found in [23–26,32]. f_i^{eq} is the equilibrium distribution function with the third order expansion in $\boldsymbol{u}$,

$$f_i^{\mathrm{eq},\alpha}(\rho^\alpha, \boldsymbol{u}) = \rho^\alpha w_i \left[1 + \frac{\boldsymbol{c}_i\cdot\boldsymbol{u}}{T_0} + \frac{(\boldsymbol{c}_i\cdot\boldsymbol{u})^2}{2T_0^2} - \frac{\boldsymbol{u}^2}{2T_0} + \frac{(\boldsymbol{c}_i\cdot\boldsymbol{u})^3}{6T_0^3} - \frac{\boldsymbol{c}_i\cdot\boldsymbol{u}}{2T_0^2}\boldsymbol{u}^2\right] \tag{7}$$