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Discrete simulation of fluid dynamics

On the stability of a relative velocity lattice Boltzmann scheme for compressible Navier–Stokes equations

Stabilité d'un schéma de Boltzmann sur réseau à vitesse relative appliqué aux équations de Navier–Stokes

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ABSTRACT

This paper studies the stability properties of a two-dimensional relative velocity scheme for the Navier–Stokes equations. This scheme inspired by the cascaded scheme has the particularity to relax in a frame moving with a velocity field function of space and time. Its stability is studied first in a linear context then on the nonlinear test case of the Kelvin–Helmholtz instability. The link with the choice of the moments is put in evidence. The set of moments of the cascaded scheme improves the stability of the d'Humières scheme for small viscosities. On the contrary, a relative velocity scheme with the usual set of moments deteriorates the stability.

RÉSUMÉ

Cet article étudie la stabilité d'un schéma de Boltzmann sur réseau à vitesse relative appliqué aux équations de Navier–Stokes bidimensionnelles. Ce schéma est une extension du schéma en cascade et s'y ramène dans un repère associé à un champ de vitesse fonction de l'espace et du temps. Sa stabilité est d'abord étudiée dans un cadre linéaire puis pour le cas test non linéaire de l'instabilité de Kelvin–Helmholtz. L'importance du choix des moments est mise en évidence. Le choix de moments du schéma en cascade améliore la stabilité de la variante de d'Humières du schéma de Boltzmann sur réseau dans le cas de petites viscosités. Au contraire, un régime de vitesse relative avec le jeu habituel des moments détériore la stabilité.

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0. Introduction

The lattice Boltzmann schemes have been successfully used for the simulation of the compressible Navier–Stokes equations in two or three dimensions [1–4]. This method aims to mimic the microscopic behaviour in order to simulate some macroscopic problems. The algorithm consists in evaluating some particle distributions. The particles, moving from node to node of a lattice, undergo a phase of collision and a phase of transport. Different collision operators have been proposed for the simulation of the Navier–Stokes equations. The simplest one is the single relaxation time operator [5,1–3] also called BGK. An alternative called the multiple relaxation times (MRT) operator [4,6] has been proposed. During the collision, some moments, linear combinations of the particle distributions, relax towards the equilibrium with a priori different velocities. It contains the particularity to offer more degrees of freedom to fix the different parameters as the viscosities. The multiple relaxation times approach is thus more flexible than the BGK. Both schemes have been well studied particularly in terms of stability [6]. They still encounter some instability features as the viscosities tend to zero that limits high Reynolds number simulations.

In 2006, a cascaded scheme improving the stability for low viscosities has been presented [7]. Its relaxation occurs in a frame moving with the fluid velocity. To understand the positive features of this scheme, a general notion of relative velocity schemes was defined [9]. Their relaxation is made for a set of moments depending on a velocity field function of space and time that is the velocity fluid for the cascaded scheme [9] and zero for the d'Humières scheme [4]. These relative velocity schemes are not restricted to the simulation of the Navier–Stokes equations: they are defined for an arbitrary number of conservation laws. Their consistency has already been studied for one and two conservation laws [8,9] but the same reasoning holds for an arbitrary number of conservation laws. The purpose is not to compare different schemes in terms of stability but to study the stability of the class of the relative velocity schemes according to the choice of some parameters.

The purpose of this contribution is to present some numerical stability results of the two dimensional nine velocities (D_2Q_9) relative velocity scheme for the compressible Navier–Stokes equations. We want to characterize the influence of the relative velocity and the link with the moments choice: the polynomials defining the moments of the cascaded scheme are different from the usual ones and may act on the stability. In a first part, we recall the basis of the relative velocity schemes. We then present the relative velocity D_2Q_9 we are interested in. The second part exhibits the results of stability, first in a linear context (L^2 von Neumann notion) and then for a non-linear test case, the Kelvin–Helmholtz instability. It puts in evidence the link between the relative velocity, the choice of the polynomials defining the moments and the stability.

1. Description of the scheme

We first introduce the relative velocity scheme for an arbitrary number of dimensions and velocities. We then particularize it to the case of two dimensions and nine velocities.

1.1. The relative velocity D_dQ_q scheme

This section presents the derivation of the relative velocity lattice Boltzmann schemes introduced in [9] and inspired by the cascaded scheme [7]. Let $\mathcal{L}$ be a Cartesian lattice in d dimensions with a typical mesh size Δx. The time step Δt is linked to the space step by the acoustic scaling $\Delta t = \Delta x/\lambda$ for $\lambda \in \mathbb{R}$ the velocity scale. We introduce $\boldsymbol{V} = (\boldsymbol{v}_0, \dots, \boldsymbol{v}_{q-1})$ a set of q velocities of $\mathbb{R}^d$. This defines the scheme called D_dQ_q. We assume that for each node $\boldsymbol{x}$ of the lattice $\mathcal{L}$, and each $\boldsymbol{v}_j$ in $\boldsymbol{V}$, the point $\boldsymbol{x} + \boldsymbol{v}_j\Delta t$ is still a node of $\mathcal{L}$. The D_dQ_q scheme computes a particle distribution $\boldsymbol{f} = (f_0, \dots, f_{q-1})$ on the lattice $\mathcal{L}$ at discrete values of time. An iteration of the scheme consists in two phases: the relaxation that is nonlinear and local in space, and the linear transport solved exactly by a characteristic method.

The relaxation phase reads more easily in a moments basis using the d'Humières framework [4]. A velocity field $\widetilde{\boldsymbol{u}}(\boldsymbol{x}, t)$ that depends on space and time being given, we define the change of basis matrix $\boldsymbol{M}(\widetilde{\boldsymbol{u}})$, usually called "matrix of moments", by

$$M(\widetilde{\boldsymbol{u}})_{kj} = P_k(\boldsymbol{v}_j - \widetilde{\boldsymbol{u}}), \qquad 0 \leqslant k, j \leqslant q-1,$$

where $(P_0, \dots, P_{q-1})$ are some polynomials of $\mathbb{R}[X_1, \dots, X_d]$. This matrix of moments, supposed to be invertible, defines the moments $\boldsymbol{m}(\widetilde{\boldsymbol{u}}) = (m_0(\widetilde{\boldsymbol{u}}), \dots, m_{q-1}(\widetilde{\boldsymbol{u}}))$ by the relation

$$\boldsymbol{m}(\widetilde{\boldsymbol{u}}) = \boldsymbol{M}(\widetilde{\boldsymbol{u}})\,\boldsymbol{f}, \tag{1}$$

where $m_k(\widetilde{\boldsymbol{u}})$ is the kth moment. If the vector $\boldsymbol{f}$ represents the coordinates of the state in the canonical basis, then the vector $\boldsymbol{m}(\widetilde{\boldsymbol{u}})$ represents the same state in the new basis obtained by the linear transformation given by $\boldsymbol{M}(\widetilde{\boldsymbol{u}})$.

The collision phase is viewed as the relaxation of the particle distributions towards an equilibrium distribution $\boldsymbol{f}^{\mathrm{eq}}$ independent of the moments and of the velocity field parameter $\widetilde{\boldsymbol{u}}$. This allows us to define the moments at equilibrium $\boldsymbol{m}^{\mathrm{eq}}(\widetilde{\boldsymbol{u}}) = (m_0^{\mathrm{eq}}(\widetilde{\boldsymbol{u}}), \dots, m_{q-1}^{\mathrm{eq}}(\widetilde{\boldsymbol{u}}))$ by

$$\boldsymbol{m}^{\mathrm{eq}}(\widetilde{\boldsymbol{u}}) = \boldsymbol{M}(\widetilde{\boldsymbol{u}})\boldsymbol{f}^{\mathrm{eq}}. \tag{2}$$

The relative velocity schemes use a diagonal relaxation phase in this shifted moments basis