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Mechanical buckling of a functionally graded cylindrical shell with axial and circumferential stiffeners using the third-order shear deformation theory

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A R T I C L E I N F O

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A B S T R A C T

This paper deals with an analytical approach of the buckling behavior of a functionally graded circular cylindrical shell under axial pressure with external axial and circumferential stiffeners. The shell properties are assumed to vary continuously through the thickness direction. Fundamental relations and equilibrium and stability equations are derived using the third-order shear deformation theory. The resulting equations are employed to obtain the closed-form solution for the critical buckling loads. A simply supported boundary condition is considered for both edges of the shell. The comparison of the results of this study with those in the literature validates the present analysis. The effects of material composition (volume fraction exponent), of the number of stiffeners and of shell geometry parameters on the characteristics of the critical buckling load are described. The analytical results are compared and validated using the finite-element method. The results show that the inhomogeneity parameter, the geometry of the shell and the number of stiffeners considerably affect the critical buckling loads.

1. Introduction

Stiffened cylindrical shells have found widespread use in modern engineering, especially in aircraft and spacecraft industry. There have been many studies on the stability of cylindrical shells, but closed-form solutions are possible only for the case when all edges are simply supported. Due to the increasing demands for high structural performances, the study of functionally graded materials in structures has received considerable attention in recent years. The buckling and postbuckling of cylindrical shells under combined loading of external pressure and axial compression have been demonstrated by Shen and Chen [1], who studied the interaction of local and overall buckling in stiffened plates and cylindrical shells.

Classical theories developed for thin elastic shells are mostly based on the Love–Kirchhoff assumptions. This theory considers that straight lines normal to the undeformed middle surface remain straight and normal to the deformed middle surface, that the normal stresses perpendicular to the middle surface can be neglected in stress–strain relationships, and that the transverse displacement is independent of the thickness coordinate. Therefore, transverse shear strains are neglected, as reported in surveys of classical shell theories by Naghdi [2] and Bert [3].

These theories are expected to produce accurate results when the thickness-to-radius ratio (h/a) is small. The application of such theories to thick or moderately thick or laminated composite shells can lead to serious errors in deflection or

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stresses. The effect of transverse shear and normal stresses in shells has been studied by Reissner [4]. The effect of transverse shear deformation was also considered by Vinson [5], Dong et al. [6,7], and Reddy [8] extended Vlasov's theory to laminated composite plates and shells. This higher-order theory based on five degrees of freedom (same number as in a first-order shear-deformation theory by Reddy [9]). This theory assumes a constant transverse deflection through the thickness and the displacements of the middle surface expanded as cubic functions of the thickness coordinate. The displacement field leads to a parabolic distribution of the transverse shear stresses and zero transverse normal strain. Therefore, no shear correction was used.

Stiffened cylindrical shells have found widespread use in modern engineering, especially in aircraft and spacecraft industry. There have been many studies on the stability of cylindrical shells, but closed-form solutions are possible only for the case when all edges are simply supported. Due to the increasing demands of high structural performances, the study of functionally graded materials in structures has received considerable attention in recent years.

The buckling and post-buckling of cylindrical shells under the combined loading of external pressure and axial compression has demonstrated by Shen and Chen [10]. An instability analysis of stiffened cylindrical shells under hydrostatic pressure was given by Barush and Singer [11]. The post-buckling of stiffened cylindrical shells under combined external pressure and axial compression was investigated by Shen et al. [12]. Using a novel finite-elements model, Sridharan and Zeggane [13] studied interacting local and overall buckling in stiffened plates and cylindrical shells. Ji Zhen and Yeh Kei [14] studied the part of the cylindrical shell which is stiffened with stringers; it is treated as an isotropic shell, and the part stiffened with rings as a discrete shell element. Based on the Donnell equations, the stability equation of nonhomogeneous cylindrical stiffened shells was obtained by use of the perturbation technique. Sadeghifar et al. [15] studied new buckling results for laminated stiffened cylindrical shells with nonuniform stringers. They used the first-order shear deformation theory to obtain the basic equations.

In this paper, the stability of FG cylindrical shells with axial and circumferential stiffeners was studied. The material properties were assumed to vary smoothly through the shell thickness according to a power-law distribution of the volume fraction of constituent materials. Initially, the stability equations were derived from the third-order shear deformation theory (TSDT). The resulting equations were employed to obtain the critical buckling loads. Also the effects of geometrical parameters, of the number of stiffeners and of the FG power index on the critical buckling load have been studied.

2. Formulation

Consider an FG cylindrical shell which is stiffened by external axial and circumferential stiffeners as shown in Fig. 1.

Throughout the current investigation, x, y and z coordinates coincide with the directions of the length, circumference and thickness of the FG cylinder, respectively.

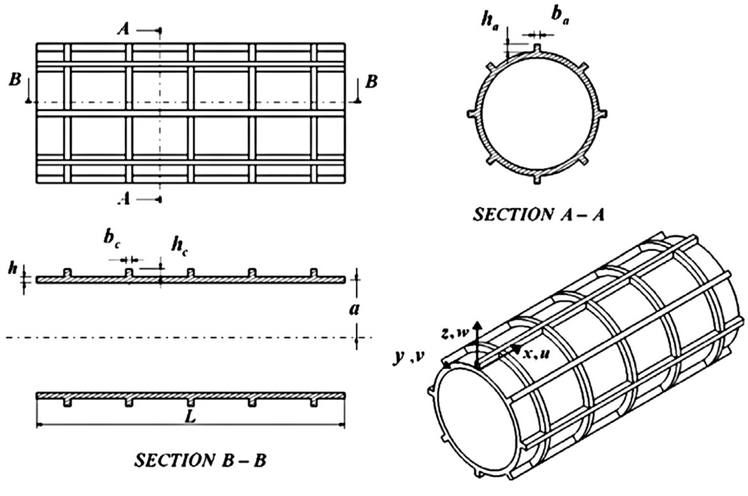

Fig. 1. Geometry and coordinate system of the stiffened cylindrical shell.

The FGM shell being made of a combined ceramic–metal material, the material distribution governed by the equation:

$$V_{\mathrm{m}}(z) = \left(\frac{1}{2} + \frac{z}{h}\right)^{\xi}, \qquad V_{\mathrm{c}}(z) = 1 - V_{\mathrm{m}}(z) \tag{1}$$

where $V(z)$ is the volume fraction of a constituent material, ξ is a non-negative volume fraction exponent, and subscripts c and m stand for ceramic and metal. Thus, the effective Young modulus of the shell assumed to vary as a power law of the thickness coordinate: