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A direct approach for continuous topology optimization subject to admissible loading

Une approche directe pour l'optimisation de la topologie continue sous chargement admissible

Zied Kammoun [a,*], Hichem Smaoui [b]

[a] *Université de Tunis El Manar, École nationale d'ingénieurs de Tunis, LR11ES16, Laboratoire de matériaux, optimisation et énergie pour la durabilité, B.P. 37, 1002, Tunis-Belvédère, Tunisia*
[b] *Salman bin Abdulaziz University, College of Engineering, P.O. Box 655, Al-Kharj 11942, Saudi Arabia*

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A B S T R A C T

In the present paper, a method is proposed for topology optimization of continuum structures subject to static and plastic admissibility conditions relative to a prescribed load. A key feature of the method is that, using a finite-element discretization, the form of the resulting topology optimization problem is similar to that of the direct static approach of the limit analysis problem. The proposed method is formulated in plane strain using Tresca materials and is illustrated on example problems taken from the literature.

R É S U M É

Dans le présent article, une méthode est proposée pour l'optimisation de la topologie des milieux continues soumis à des conditions d'admissibilité statique et plastique relativement à un chargement imposé. Une propriété essentielle de la méthode est qu'en utilisant une discrétisation par éléments finis, la forme du problème d'optimisation de la topologie résultant est similaire à celle du problème direct de l'analyse limite formulé selon l'approche statique. La méthode proposée est formulée en déformations planes en utilisant un matériau de Tresca. Elle est illustrée à travers des exemples de problèmes issus de la littérature.

1. Introduction

Research in topology optimization of continuum structures has witnessed a considerable development during the last decades [1–5]. This development led to near maturity, as demonstrated by the numerous successful applications in industry [6] and the emergence of powerful dedicated topology optimization software [7]. It is noted, however, that most of the work on continuum topology optimization has been restricted to linear elastic material behavior. Elastic design is historically the

* Corresponding author.
E-mail addresses: kammounzied@yahoo.fr (Z. Kammoun), hismaoui@yahoo.fr (H. Smaoui).

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most common and most demanded type of design, and continuum topology design is not an exception in this regard. Elastoplastic analyses that seek to determine response quantities, especially evolution methods, are known for their high computational demand. On the contrary, direct methods of limit analysis are known to require lower computational effort to determine limit states in terms of either stress field or displacement/velocity field solutions. In an automated design context, where computational efficiency is a primary factor, direct methods of limit analysis become attractive for their considerable computational saving potential. Member sizing optimization of specific types of structures, e.g., trusses and frames, subject to plastic design constraints has occasionally been treated in the literature using direct methods of plastic collapse analysis [8]. Nevertheless, topology design of continuum media, involving this type of analyses, is nearly inexistent in the literature. Some research has dealt with continuum topology design of nonlinear elastic structures where the tools developed for the linear behavior were adapted and extended to the nonlinear case [2] and only a few tentatives were directed to design involving elastoplastic [9] or plastic analyses [10].

The present work is precisely concerned with the integration of direct methods of limit analysis into a methodology for plastic topology design of continuum structures. A judicious formulation for this class of problems is proposed such that the continuous design problem, which is expressed in terms of continuous material densities as design variables, takes on a mathematical form similar to that of a direct limit analysis problem. The computational demand for the topology design problem using the proposed approach is consequently expected to be in the order of that of the execution of a single limit analysis. In the present paper, the topology design problem of plastic continuum structures is formulated according to the microscopic, material approach and based on direct limit analysis. Some of the desirable properties of the design problem expressed in terms of continuous densities are highlighted. Formulation in these design variables leads to the so-called continuous or porous topologies. It has often been proposed as a preliminary to the ultimate goal of producing optimal black and white, i.e. 0–1, topologies. Owing to the recent breakthroughs in material technology, though, it is becoming possible to tailor the microstructure to suit a wide range of desired material properties and gradients. This development has regenerated interest in continuous topologies. Finally, a number of example design problems are treated to illustrate the capabilities of the proposed method and to compare the designs it generates with those produced using existing methods.

2. The static method of limit analysis

The following terminology defined in [11,12] will be adopted in the present paper. A stress field σ is said to be statically admissible (SA) if field equilibrium equations, stress vector continuity, and stress boundary conditions are satisfied. It is said to be plastically admissible if $f(\sigma) \leqslant 0$, where $f(\sigma)$ is the plasticity criterion of the material. A stress field σ that is both SA and plastically admissible will be said to be fully admissible or simply "admissible". A loading system $Q \in \mathbb{R}^{\mathrm{n}}$ in equilibrium with a statically admissible stress field σ, $Q = Q(\sigma)$, is said to be admissible. The n components of Q are called loading parameters. The relationship $Q = Q(\sigma)$, which usually describes either field equilibrium equations, when body forces are present, or boundary conditions on the stress vector, is linear in both cases. A solution of the limit analysis problem relative to the ith loading parameter is found by solving the following optimization problem for an admissible stress field σ such that:

$$
\begin{aligned}
&Q_{\mathrm{lim}} = \left(Q_1^{\mathrm{d}}, \ldots, \lambda_0 Q_i^{\mathrm{d}}, \ldots, Q_n^{\mathrm{d}}\right)\\
&\quad \lambda_0 = \max\left\{\lambda,\ Q(\sigma) = \left(Q_1^{\mathrm{d}}, \ldots, \lambda Q_i^{\mathrm{d}}, \ldots, Q_n^{\mathrm{d}}\right)\right\}
\end{aligned} \tag{1}
$$

where Q^{d} is a specified admissible loading. The resulting loading $Q(\sigma)$ is a limit loading of the mechanical domain. This formulation defines the static, lower bound problem of limit analysis, as it will be dealt with in the present work. Unlike the usual response-oriented analysis methods, the lower bound method of limit analysis determines the stress field at the limit state only. It provides neither information on the stress field at the intermediate stages of the loading process nor on the kinematic quantities at any loading step. For this reason, the method is classified as a direct method. This lack of information is compensated by lower computational demand which, in case the missing information is not necessary, becomes a paramount advantage. Another merit of the static approach is the status of rigorous lower bound of the limit load.

3. Finite-element formulation of the static problem

The numerical plane strain formulation of the static, lower bound problem is described in detail in [13]. Consider a triangular finite element discretization of the mechanical domain Ω in the global reference frame (x, y). The stress field is assumed to be linear in x and y within the element. Across interelement boundaries, it can be discontinuous, provided the stress vector acting on the element boundary remains continuous. In plane strain, the Tresca criterion is written as:

$$
f(\sigma) = \sqrt{(\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2} - 2s \leqslant 0 \tag{2}
$$

or equivalently as:

$$
S(\sigma) = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \leqslant s \tag{3}
$$