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Solving the vibration problem of inhomogeneous orthotropic cylindrical shells with hoop-corrugated oval cross section

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ABSTRACT

Based on the framework of Flügge's shell theory, transfer matrix approach and Romberg integration method, this paper investigates how corrugation parameters and material homogeneity affect the vibration behavior of isotropic and orthotropic oval cylindrical shells with sine-shaped hoop. Assume that the Young's moduli, shear moduli and density of the orthotropic material are continuous functions of the coordinate in the circumferential direction. The governing equations of non-homogeneous, orthotropic oval cylindrical shells with variable homogeneity along its circumference are derived and put in a matrix differential equation as a boundary-value problem. The trigonometric functions are used with Fourier's approach to approximate the solution in the longitudinal direction, and also to reduce the two-dimensional problem to a one-dimensional one. Using the transfer matrix approach, the equations can be written in a matrix differential equation of the first order and solved numerically as an initial-value problem. The proposed model is applied to get the vibration frequencies and mode shapes of the symmetric and antisymmetric vibration modes. The sensitivity of the vibration behavior to the corrugation parameters, homogeneity variation, ovality, and orthotropy of the shell is studied for different type modes of vibration.

1. Introduction

Orthotropic shells with circular and noncircular profiles are more efficient than isotropic ones, not only because of their higher wall stiffness, but also because they are less sensitive to initial imperfections. A special type of orthotropic shells, discretely reinforced longitudinal or circumferential corrugation, is very cost effective and widely used in various areas of modern engineering applications, such as air engineering, marine structures, agricultural silos, and piping. Circumferentially corrugated shells are a type of sensitive elastic elements widely used in precision instruments and meters, and they are used in structures with low flexural stiffness in the longitudinal direction and high flexural stiffness in the circumferential direction. The vibration, bending and stability of corrugated shells should be studied systematically to obtain the entire mechanical properties to help designers to achieve a reduction in weight and an increase in stiffness. Mathematically, the consideration of non-homogeneity, orthotropy, corrugations and aspect ratios leads to a very complex problem involving several parameters. So, numerical or approximate techniques are necessary for their analysis. Since more attention is paid to the analysis of the behavior of shells of a non-uniform cross-section with inhomogeneous materials, researches on the vibration of isotropic and orthotropic circular cylindrical shells have been undertaken by many researchers, since the basic equations for this had been established by Love [1], Rayleigh [2], and Flügge [3]. The best collection of documents can be

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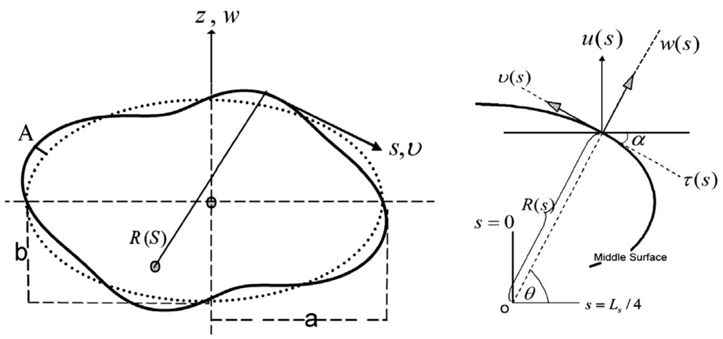

Fig. 1. Coordinate system and geometry of an inhomogeneous elastic cylindrical shell with a hoop-corrugated oval cross section.

found in Leissa [4], in which more than 500 publications are analyzed and discussed in both linear and non-linear vibration cases. Other related literatures for the vibration of homogeneous oval cylindrical shells can be found in [5–11]. On the other hand, there are a few publications that studied the vibration characteristics of the isotropic and orthotropic corrugated cylindrical shells, e.g., [12–17], whereas Tsiolkovsky [18] was the first one to see that the corrugated shells can be used in lighter-than-air engineering. As it has been found recently, a few studies have been directed toward the vibration behavior of smooth, inhomogeneous, orthotropic cylindrical shells [19–24]. Despite extensive works have been carried out, it is likely that the combined effects of corrugations, orthotropy, and non-homogeneity of materials on the vibration behavior of oval and circular cylindrical shells have not been analyzed. This paper aims at studying the combined effects of non-homogeneity of materials and corrugations of surface on the vibration behavior of elastic orthotropic shells. The basic equations of the vibration of a non-homogeneous, corrugated orthotropic oval cylindrical shell are derived from the framework of Flügge's shell theory. The transfer matrix is employed to get the vibration frequencies and the corresponding mode shapes for the symmetric- and antisymmetric-type modes. The results reveal that the variations of the oval cylindrical shell parameters, homogeneity variation, orthotropy of material and corrugation parameters have significant effects on the values of the natural frequencies and the mode shapes. The results are cited in tabular and graphical forms.

2. The model

2.1. Shell theory

It is known by researchers that the study of vibration problems in shells depends on the shell geometry and on the model of shell theory adopted. The present problem is modeled using Flügge's theory, which is very accurate in the case of thin shells due to the consideration of the shear deformation effect in the equilibrium equations.

2.2. Geometric formulation

The present shell is a corrugated orthotropic oval cylindrical shell with a sine-shaped hoop. The curvature of the cross-section profile of a corrugated oval shell is defined by the equation $r_0/R(s) = f(s)$, where R is the variable radius of curvature along the cross-section mid-line, r_0 is the reference radius of curvature, chosen to be the radius of a circle having the same circumference as the oval profile, and $f(s)$ is a prescribed function of s and can be taken as a simple form by the first two terms of the Fourier series representation of the curvature of a general cylindrical shell, see [25]; the corrugated surface can be taken as:

$$f(s) = 1 + \varepsilon \cos(2s) + A \sin(2Ns), \quad 0 \le s \le 2\pi \tag{1}$$

where A is the corrugation amplitude and N is the number of circumferential corrugations over the shell surface, as ε is the ovality parameter, which measures the eccentricity of the cross section of the shell, and to prevent negative curvature, $|\varepsilon|$ should be equal to or less than unity. Such a function describes a doubly-symmetric oval cross section and can be approximated by an ellipse. The ovality parameter ε is expressed by the semi-major, a, and semi-minor, b, axes of the oval profile given by [9]:

$$\begin{aligned} &\varepsilon = 3Q - (36/35)Q^3 \\ &Q = (A/B - 1)/(A/B + 1) \\ &B = b/r, \qquad A = a/r_0 \end{aligned} \tag{2}$$

The position of a point on the middle surface of the shell is defined by the cylindrical coordinates (x, s, z), as shown in Fig. 1. The displacements of the middle surface of the shell are denoted by u, υ and w in the axial, hoop and transverse directions, respectively. The shell geometry is described by T_0, the thickness of the shell, L_x, the axial length, and $(L_s = 2\pi r_0)$, the