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Bonding a linearly piezoelectric patch on a linearly elastic body

Analyse asymptotique d'un patch linéairement piézoélectrique lié à un corps linéairement élastique

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A B S T R A C T

A rigorous study of the asymptotic behavior of the system constituted by a very thin linearly piezoelectric plate bonded on a linearly elastic body supplies various models for an elastic body monitored by a piezoelectric patch.

R É S U M É

Une étude rigoureuse du comportement asymptotique du système constitué par une plaque linéairement piézoélectrique collée sur un corps linéairement élastique fournit divers modèles de contrôle de structures élastiques par des patches piézoélectriques.

1. Introduction

Many studies dealing with the mathematical modeling of piezoelectric devices were devoted to the behavior of the sole patches and provided various asymptotic models for thin linearly piezoelectric plates (see [1] and the references therein). However, the essential technological interest of piezoelectric patches being the monitoring of a deformable body they are bonded to, here we intend to propose various asymptotic models for *the behavior of the body* through the study of the *system* constituted by a very thin linearly piezoelectric flat patch perfectly bonded to a linearly elastic three-dimensional body. A reference configuration for the body is an open set Ω laying in $\{x_3 < 0\}$ whose part of its Lipschitz-continuous boundary $\partial\Omega$ is a non-empty domain S in $\{x_3 = 0\}$ and such that $S \times (-L, 0)$ is included in Ω for some positive real number L, while the patch occupies $B^\varepsilon := S \times (0, \varepsilon)$, ε being a small real number; let $\mathcal{O}^\varepsilon := \Omega \cup S \cup B^\varepsilon$. The body is clamped on a part Γ_0

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of $\partial\Omega \setminus S$ with a positive two-dimensional Hausdorff measure $\mathcal{H}_2(\Gamma_0)$, and subjected to body forces and surface forces on $\Gamma_1 := \partial\Omega \setminus (S \cup \Gamma_0)$ of densities f and F. Moreover, for all δ in $\mathbb{R}$, let S^δ denote $S + \delta e_3$, $\{e_1, e_2, e_3\}$ being a basis of the Euclidean physical space assimilated to $\mathbb{R}^3$, surface forces of density G acts on S^ε whilst the patch is free of mechanical loading and electric charges in B^ε and on its lateral boundary $\partial S \times (0, \varepsilon)$. If u^ε, $e(u^\varepsilon)$, σ^ε denote the fields of displacement, strain and stress in $\mathcal{O}^\varepsilon$ and φ^ε, D^ε stand for the electric potential and the electric displacement, part of the equations describing the electromechanical equilibrium read as:

$$\begin{cases} \operatorname{div}\sigma^\varepsilon = \tilde{f} \quad \text{in } \mathcal{O}^\varepsilon, \quad u^\varepsilon = 0 \quad \text{on } \Gamma_0, \quad \sigma^\varepsilon n = F \quad \text{on } \Gamma_1, \quad \sigma^\varepsilon n = G^\varepsilon \quad \text{on } S^\varepsilon, \quad \sigma^\varepsilon n = 0 \quad \text{on } \partial S \times (0, \varepsilon) \\ \operatorname{div} D^\varepsilon = 0 \quad \text{in } B^\varepsilon, \quad D^\varepsilon \cdot n = 0 \quad \text{on } \partial S \times (0, \varepsilon) \\ \sigma^\varepsilon = ae(u^\varepsilon) \quad \text{in } \Omega, \quad (\sigma^\varepsilon, D^\varepsilon) = \dfrac{1}{\varepsilon} M\big(e(u^\varepsilon), \nabla\varphi^\varepsilon\big) \quad \text{in } B^\varepsilon \end{cases} \tag{1}$$

$\tilde{f}$ is the extension of f to B^ε by 0, n is the unit outward normal and a denotes the elasticity tensor which satisfies:

$$a \in L^\infty\big(\Omega; \operatorname{Lin}(\mathbb{S}^3)\big), \qquad \exists c; c|e|^2 \leqslant a(x)e \cdot e, \quad \forall e \in \mathbb{S}^3, \text{ a.e. } x \in \Omega \tag{2}$$

where $\operatorname{Lin}(\mathbb{S}^N)$ is the space of linear operators on the space $\mathbb{S}^N$ of $N \times N$ symmetric matrices whose inner product and norm are noted $\cdot$ and $|\cdot|$ as in $\mathbb{R}^3$. If $\mathbb{H} := \mathbb{S}^3 \times \mathbb{R}^3$ is equipped with an inner product and a norm also denoted as previously, then M is an element of $L^\infty(S \times \mathbb{R}; \operatorname{Lin}(\mathbb{H}))$ independent of x_3 satisfying:

$$M = \begin{bmatrix} \alpha & -\beta \\ \beta^T & \gamma \end{bmatrix}, \qquad \exists\kappa > 0; \quad \kappa|e|^2 \leqslant \alpha(x)e \cdot e, \qquad \kappa|g|^2 \leqslant \gamma(x)g \cdot g, \quad \forall(e, g) \in \mathbb{H}, \text{ a.e. } x \in S \times \mathbb{R} \tag{3}$$

The models will be distinguished according to the additional necessary boundary conditions on S^ε and S, characterized by an index p in $\{1, 2\}^2$. Case $p_1 = 1$ corresponds to a condition for the electric displacement on S^ε:

$$D^\varepsilon \cdot n = q^\varepsilon \quad \text{on } S^\varepsilon \tag*{(4)$_1$}$$

q^ε being a density of electrical charges, while $p_1 = 2$ corresponds to a condition of given electrical potential:

$$\varphi^\varepsilon = \varphi_0^\varepsilon \quad \text{on } S^\varepsilon \tag*{(4)$_2$}$$

roughly speaking, $p_1 = 1$ deals with patches used as sensors, whereas $p_1 = 2$ concerns actuators (see [1,2]). Index p_2 accounts for the status of the interface between the patch and the body: $p_2 = 1$ corresponds to an electrically impermeable interface, $p_2 = 2$ corresponds to a grounded interface:

$$D^\varepsilon \cdot n = 0 \quad \text{on } S \tag*{(5)$_1$}$$

$$\varphi^\varepsilon = 0 \quad \text{on } S \tag*{(5)$_2$}$$

It will be convenient to use the following notations:

$$\begin{cases} \hat{k} := (\hat{e}, \hat{g}), \qquad \hat{e} := (e_{\alpha\beta})_{\alpha,\,\beta\in\{1,2\}}, \qquad \hat{g} := (g_1, g_2), \quad \forall k = (e, g) \in \mathbb{H} \\ \tilde{e} \in \mathbb{S}^3; \qquad \tilde{e}_{\alpha\beta} = e_{\alpha\beta}, \quad 1 \leqslant \alpha, \beta \leqslant 2, \qquad \tilde{e}_{i3} = 0, \quad 1 \leqslant i \leqslant 3, \forall e \in \mathbb{S}^2 \\ k(r) = k(v, \psi) := \big(e(v), \nabla\psi\big), \quad \forall r = (v, \psi) \in H^1\big(B^\varepsilon; \mathbb{R}^3\big) \times H^1\big(B^\varepsilon\big) \\ e(v) \in \mathcal{D}'\big(S; \mathbb{S}^2\big); \qquad \big(e(v)\big)_{\alpha\beta} = \dfrac{1}{2}(\partial_\alpha v_\beta + \partial_\beta v_\alpha), \quad \forall v \in \mathcal{D}'\big(S; \mathbb{R}^2\big) \end{cases} \tag{6}$$

where the same symbol $e(\cdot)$ stands for the symmetrized gradient in the sense of distributions of $\mathcal{D}'(\mathcal{O}; \mathbb{R}^3)$, $\mathcal{O} \in \{\Omega, B^\varepsilon, \mathcal{O}^\varepsilon\}$, or $\mathcal{D}'(S; \mathbb{R}^2)$. Moreover we introduce some spaces, linear and bilinear forms in order to supply a variational formulation of (1)–(5). An electromechanical state will be an element $r = (v, \psi)$ of

$$V_p := H^1_{\Gamma_0}\big(\mathcal{O}^\varepsilon; \mathbb{R}^3\big) \times \Phi_p, \quad \Phi_{(1,1)} = H^1\big(B^\varepsilon\big), \quad \Phi_{(1,2)} = H^1_S\big(B^\varepsilon\big), \quad \Phi_{(2,1)} = H^1_{S^\varepsilon}\big(B^\varepsilon\big), \quad \Phi_{(2,2)} = H^1_{S\cup S^\varepsilon}\big(B^\varepsilon\big) \tag{7}$$

where, for any domain $\mathcal{O}$ of $\mathbb{R}^3$, $H^1_\Gamma(\mathcal{O}; \mathbb{R}^3)$ and $H^1_\Gamma(\mathcal{O})$ respectively denote the subspaces of $H^1(\mathcal{O}; \mathbb{R}^3)$ and $H^1(\mathcal{O})$ of all elements with vanishing traces on a part Γ of $\partial\mathcal{O}$. One makes the following assumptions on the data:

$$\begin{cases} \varphi_0 \text{ denotes the restriction to } S \text{ of an element of } H^{1/2}\big(\{x_3 = 0\}\big) \text{ still denoted by } \varphi_0 \\ (f, F, G, q) \in L^2\big(\Omega; \mathbb{R}^3\big) \times L^2\big(\Gamma_1; \mathbb{R}^3\big) \times L^2\big(S; \mathbb{R}^3\big) \times L^2\big(S\big), \quad \displaystyle\int_S q \,\mathrm{d}\hat{x} = 0 \\ G^\varepsilon(x + \varepsilon e_3) = G(x), \qquad q^\varepsilon(x + \varepsilon e_3) = q(x), \qquad \varphi_0^\varepsilon(x + \varepsilon e_3) = \varepsilon\varphi_0(x), \qquad \text{a.e. } x \in S \end{cases} \tag{8}$$

It is well known that for all φ_0 in $H^{1/2}(\{x_3 = 0\})$, there exists an element of $H^1(S \times (-L, 0))$ when $p_2 = 1$, $H^1_{S_{-L}}(S \times (-L, 0))$ when $p_2 = 2$, still denoted by φ_0 whose trace on S is φ_0. Hence the element $\varphi^\varepsilon_{o,p}$ of Φ_p defined by $\varphi^\varepsilon_{o,p}(x) = \varepsilon\varphi_0(\hat{x}, (x_3 - \varepsilon)L/\varepsilon)$ satisfies: