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Flexural vs. tensile strength in brittle materials

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ABSTRACT

The tests leading to the determination of the strength of brittle materials show a very wide scattering and a noticeable difference between flexural and tensile strengths. The corresponding statistics are usually described by the Weibull law, which only partly explained the observed difference. From a theoretical point of view, the coupled criterion reaches the same conclusion, the flexural strength is higher than the tensile one. It is shown that these two approaches complement to give a satisfying explanation of the difference between the flexural and tensile strengths. Moreover, according to the coupled criterion, the tensile strength appears to be the only material parameter.

1. Introduction

Determining the strength of a brittle material is based on tensile or flexural tests on unnotched specimens (Fig. 1) [1].

The role of randomly distributed defects is decisive, leading to a large scattering in the measurements. The statistics are usually described by the Weibull law [2]. It leads to a significant difference between the measurements made in bending and tension. The flexural strength is higher than the tensile one. Indeed, for two samples of the same size, only one half of the sample is stressed in bending while the whole is in tension, then fewer defects are involved in bending. Nevertheless, the Weibull law often underestimates the flexural strength.

From a theoretical point of view, in fracture mechanics, the crack nucleation cannot be predicted by the well-known Griffith law. This latter only allows deciding whether a *pre-existing* crack can grow or not. The problem of the initiation of a new crack in brittle materials has been the subject of many studies since the 70s. They enter into a theory baptized, since the end of the 90s, Finite Fracture Mechanics [3,4]. Among them, the coupled criterion proposed in 2002 [5] seems to be one of the most promising and has proven its effectiveness in particular to predict the failure of v-notched specimens. It is based on the simultaneous fulfilment of a stress and an energy conditions. It leads to a similar conclusion on the ratio between the flexural and the tensile strengths.

The aim of this work is to analyze both approach and to show that they are not contradictory. They even complement to give a satisfying explanation of the difference between the flexural and tensile strengths.

2. The statistical Weibull model

In brittle materials, the Weibull law [2,6,7] provides a statistical approach of the failure of a specimen of volume V undergoing a uniaxial stress field σ. It relies on the theory of the weakest link. Under a uniform tension $\sigma = \bar{\sigma} =$ Constant, the failure probability is

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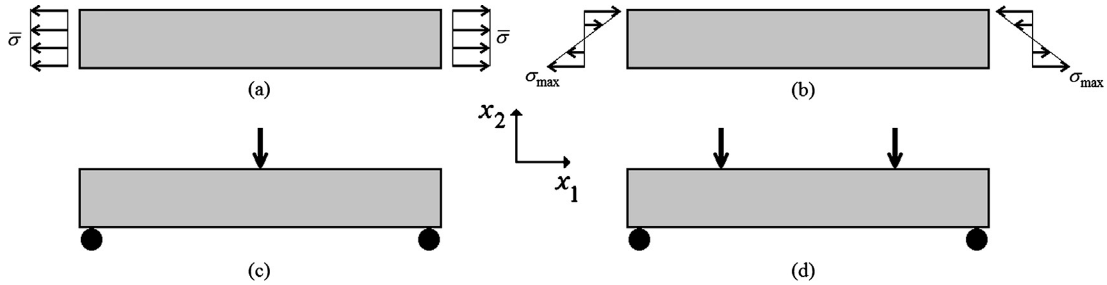

Fig. 1. Four different loadings to measure the strength of a brittle material: (a) uniform tension, (b) pure bending, (c) 3-point bending, (d) 4-point bending.

$$P_T(\sigma, V) = 1 - \exp\left(-\left(\frac{\bar{\sigma}}{\sigma_0}\right)^m \frac{V}{V_0}\right) \tag{1}$$

Where m is the Weibull modulus and σ_0, V_0 a pair of scaling parameters such that

$$P_T(\sigma_0, V_0) = 1 - \frac{1}{e} \simeq 0.632 \tag{2}$$

Here and further in the document, the indices "T, B, PB, 3P-B, 4P-B" hold respectively for tension, bending, pure bending, 3-point bending and 4-point bending.

For a uniaxial but non-uniform stress field σ, the Weibull law takes the following form

$$P(\sigma, V) = 1 - \exp\left(-\left(\frac{\sigma_W}{\sigma_0}\right)^m\right) \quad \text{with } \sigma_W = \frac{1}{V_0^{1/m}}\left[\int_{V'} \sigma^m \mathrm{d}V\right]^{1/m} \tag{3}$$

Where V' (involved in the integral) is the tested volume of the specimen.

For a pure bending loading, the tensile stress linearly decreases through the thickness from a maximum value σ_{max} on the face of the specimen under tension to $-\sigma_{max}$ on the opposite face under compression

$$\sigma(x) = \sigma_{max}\left(1 - \frac{2x_2}{h}\right) \tag{4}$$

Where h is the thickness of the specimen.

Thus, in a pure bending loading, only one half of the specimen of volume V (see (1)) is under tension and V' is the volume defined by $0 \le x_2 \le h/2$, as a consequence (3) gives

$$\sigma_W = \frac{\sigma_{max}}{[2(m+1)]^{1/m}} \quad \text{and} \quad P_{PB}(\sigma) = 1 - \exp\left(-\left(\frac{\sigma_W}{\sigma_0}\right)^m\right) \tag{5}$$

Then the ratio R_{PB} between the flexural σ_c^{PB} and the tensile σ_c^T strengths for a pure bending can be obtained considering

$$P_T(\sigma, V) = P_{PB}(\sigma, V) \Rightarrow R_{PB} = \frac{\sigma_c^{PB}}{\sigma_c^T} = \frac{\sigma_{max}}{\bar{\sigma}} = [2(m+1)]^{1/m} \tag{6}$$

Similar relations can be derived for the 3- and 4-point bending tests [8]

$$R_{3P\text{-}B} = \left[2(m+1)^2\right]^{1/m}; \qquad R_{4P\text{-}B} = \left[\frac{6(m+1)^2}{m+3}\right]^{1/m} \tag{7}$$

Surprisingly, the ratio R in (6) and (7) does not depend on the specimen thickness h.

Fig. 2 shows a comparison between (6) and (7) for various Weibull modulus m. A noticeable property of these three curves is that $R \to 1$ as $m \to \infty$, i.e. as the scattering decreases and finally vanishes, the law becoming entirely deterministic.

Data from manufacturers are available for different materials (E is the Young modulus, ν the Poisson ratio, K_{Ic} the toughness of the material):

- Alumina AD998 [9], $E = 370$ GPa, $\nu = 0.22$, $K_{Ic} = 4.5$ MPa $\sqrt{}$m, $\sigma_c^T = 248$ MPa, $\sigma_c^B = 375$ MPa, $m = 21$ [10].
- Polymer PR520 [11,12], $E = 4$ GPa, $\nu = 0.4$, $K_{Ic} = 2.2$ MPa $\sqrt{}$m, $\sigma_c^T = 82.1$ MPa, $\sigma_c^B = 153.1$ MPa, $m = 16$ [8].
- Silicon carbide SA [13], $E = 430$ GPa, $\nu = 0.14$, $K_{Ic} = 4.6$ MPa $\sqrt{}$m, $\sigma_c^T = 234$ MPa, $\sigma_c^{4P\text{-}B} = 380$ MPa, $\sigma_c^{3P\text{-}B} = 550$ MPa, $m = 10$.