C. R. Mecanique 343 (2015) 199–209

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

www.sciencedirect.com

Numerical limit analysis and plasticity criterion of a porous Coulomb material with elliptic cylindrical voids

Franck Pastor [a], Joseph Pastor [b], Djimedo Kondo [c,*]

[a] Athénée royal Victor-Horta, rue de la Rhétorique, 16, Bruxelles, Belgium
[b] Laboratoire LOCIE, UMR 5271 CNRS, Université de Savoie, 73376 Le Bourget-du-Lac, France
[c] Institut Jean-Le-Rond-D'Alembert, UMR 7190 CNRS, UPMC, 75252 Paris cedex 05, France

ARTICLE INFO

Article history:
Received 12 October 2014
Accepted 8 December 2014
Available online 31 January 2015

Keywords:
Gurson-type models
Cylindrical voids
Porous Coulomb material
Micromechanics
Limit analysis
Static and mixed methods
Conic programming

ABSTRACT

The paper is devoted to a numerical Limit Analysis of a hollow cylindrical model with a Coulomb solid matrix (of confocal boundaries) considered in the case of a generalized plane strain. To this end, the static approach of Pastor et al. (2008) [18] for Drucker–Prager materials is first extended to Coulomb problems. A new mixed—but rigorously kinematic—code is elaborated for Coulomb problems in the present case of symmetry, resulting also in a conic programming approach. Owing to the good conditioning of the resulting optimization problems, both methods give very close bounds by allowing highly refined meshes, as verified by comparing to existing exact solutions. In a second part, using the identity of Tresca (as special case of Coulomb) and von Mises materials in plane strain, the codes are used to assess the corresponding results of Mariani and Corigliano (2001) [13] and of Madou and Leblond (2012) [11] for circular and elliptic cylindrical voids in a von Mises matrix. Finally, the Coulomb problem is investigated, also in terms of projections on the coordinate planes of the principal macroscopic stresses.

1. Introduction

In his celebrated paper [1], Gurson has given an approximate expression of the plasticity criterion of porous materials based on the consideration of a hollow von Mises sphere or cylinder and on the kinematic method of the Limit Analysis (LA) theory. Recent studies were devoted to porous materials with a matrix exhibiting a pressure-sensitive behavior ([2–4], etc.). Other extensions of the Gurson model accounting for void-shape effects have been also proposed: see among others the references [5–10] for spheroidal voids, and [11,12] for ellipsoidal cavities.

In the case of cylindrical cavities with circular or elliptic cross-sections, up to our knowledge, in the literature only exist the theoretical studies of [13] and [11] for porous materials with von Mises matrix. From the numerical point of view, the static and kinematic methods of LA have been elaborated for Gurson problems, i.e. a von Mises matrix with cylindrical cavities, in the references [14–16], and [17]. A first attempt to extend these works for pressure-dependent criteria was presented in [18] for porous Drucker–Prager materials under a generalized plane strain. Analogous studies for porous Tresca–Coulomb materials with cylindrical voids have not been considered in the literature, no more than theoretical investigations, up to our knowledge.

* Corresponding author.
E-mail addresses: F.pastor@skynet.be (F. Pastor), joseph.pastor@univ-savoie.fr (J. Pastor), djimedo.kondo@upmc.fr (D. Kondo).

http://dx.doi.org/10.1016/j.crme.2014.12.004

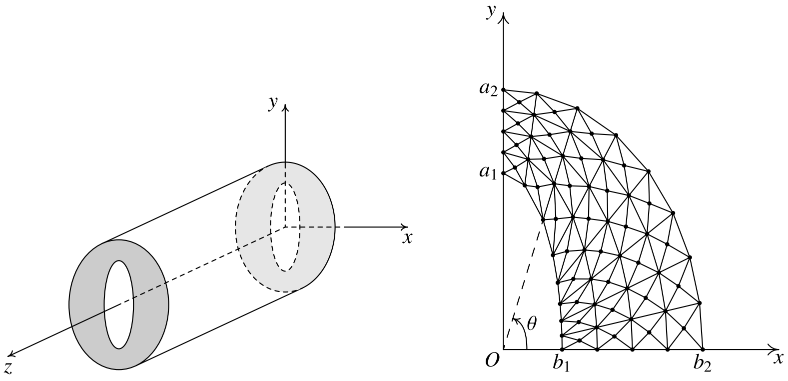

Fig. 1. The hollow cylinder model ($a_1/b_1 = 3$, $f = 0.2$).

Therefore, the main purpose of the present paper is to provide lower and upper bounds to be used as reference values for forthcoming attempts to determine approximate criteria for porous Tresca–Coulomb materials with circular and elliptic cylindrical cavities. An example of the usefulness of these bounds can be found in a recent paper [19], where the previous numerical results of [4] were used for comparison and validation of the proposed estimates for the macroscopic criterion.

The paper is organized as follows. First, we briefly present the hollow cylindrical problem and its formulation in terms of limit analysis. Then, we recall, briefly, the limit analysis methods used here, and the corresponding expressions needed to assess the result through a complete post-analysis of the optimal fields. The next step is devoted to detail the proposed numerical formulation for the mixed (but rigorously kinematic) method, focusing on the case of the generalized plane strain, since it is the first time that the LA mixed approach is applied to mechanical problems in this case of symmetry.

Taking advantage of the identity of Tresca and von Mises materials in plane strain, the static and kinematic codes are firstly validated by comparison with the Gurson exact solution to the hollow cylinder model under transverse isotropic loadings. In a second step, for these materials and for circular and elliptic cylindrical problems, the codes are used to assess the available results of [13] and [11] in terms of projections on the coordinate planes of the macroscopic principal stresses. Finally, the Coulomb problem is investigated, also for circular and elliptic cross-sections of the cylindrical void.

2. The hollow cylinder model

The hollow cylinder model is made up of an elliptic cylindrical cavity embedded in a cylinder with a confocal boundary. The solid matrix is an isotropic, homogeneous and rigid-plastic Coulomb material, with the Tresca material as a special case. Fig. 1 presents the geometric model, where the given aspect ratio a_1/b_1 and porosity f allow us to determine the parameters a_2 and b_2 of the confocal elliptic boundary.

In the generalized plane strain problem, the stress tensor σ and the strain rate tensor d do not depend on the coordinate z. Let us note Σ and D the macroscopic stress and strain rate tensors; these quantities are related to the local fields by the averages over the cross-section S of the model in the plane (x, y):

$$\Sigma_{ij} = \frac{1}{S}\int\limits_{S} \sigma_{ij}\,\mathrm{d}S; \qquad D_{ij} = \frac{1}{S}\int\limits_{\partial S} \frac{1}{2}(u_i n_j + u_j n_i)\,\mathrm{d}(\partial S) \tag{1}$$

where u denotes the velocity vector and n the (outward) normal unit vector to the boundary ∂S of the model.

More precisely, the local velocity vector u is defined by the components $u_x(x, y), u_y(x, y), u_z = D_z z$, so that the axis z is a principal axis for both tensors. This is a special case of the general 3D-plane problem proposed in [20] from specific periodicity considerations, recently used in [21], among others.

Under the uniform strain rate boundary conditions, i.e. $u_i = D_{ij}x_j$ (in which x represents the position vector), on the outer boundary ∂S, the overall virtual dissipated power P_{tot} can be written as follows:

$$P_{\text{tot}} = S\,Q \cdot q \tag{2}$$

where the loading vector Q and the generalized velocity q are defined as:

$$Q_1 = \Sigma_x, \quad Q_2 = \Sigma_y, \quad Q_3 = \Sigma_z, \quad Q_4 = \Sigma_{yz}, \quad Q_5 = \Sigma_{zx}, \quad Q_6 = \Sigma_{xy}$$
$$q_1 = D_x, \quad q_2 = D_y, \quad q_3 = D_z, \quad q_4 = 2D_{yz}, \quad q_5 = 2D_{zx}, \quad q_6 = 2D_{xy}$$

From the matrix isotropy and the elliptic cross-section of the model, the resulting material is orthotropic with the axis z as one anisotropy axis, becoming transversally isotropic around this axis for circular cross-sections. Here the macroscopic criterion $g(\Sigma)$ is investigated in the $(Oxyz)$ anisotropy frame in terms of projection on the loading planes. For example, we search for the projection of $g(\Sigma)$ in the (Q_1, Q_2) plane by optimizing Q_2 for fixed Q_1, the other loading components being