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The transition to turbulence in parallel flows: A personal view

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ABSTRACT

This is a discussion of the present understanding of transition to turbulence in parallel flows, based upon the idea that it arises from a subcritical instability. The result is a coupled set of equations, one amplitude equation in the direction of translational invariance of the geometry coupled with the standard Reynolds equation for the average transfer of momentum. It helps to understand a basic feature of the transition in parallel flows, namely that turbulence manifests itself in localised domains growing at a constant speed depending on the Reynolds number.

1. Historical introduction

The transition to turbulence is observed in flows when their speed with respect to the walls increases beyond a certain limit. This transition is generally attributed to the fact that, beyond this critical speed, the flow becomes unstable, an idea that can be traced back to a founding paper in fluid mechanics by Osborne Reynolds in 1883 [1]. How important it was, this paper was not written very clearly and its conclusions were somewhat ambiguous. Perhaps this explains why part of its message has been more or less forgotten over the years. To take an example, by reading a review on the transition in parallel flows [2] one sometimes finds the word “instability” in tentative explanations of the occurrence of localised structures, but without any clear definition of what is meant there. To explain that a fluid crosses the boundary of a turbulent spot in both the laminar-to-turbulent and turbulent-to-laminar directions Coles writes that there should be “some kind of strong local instability in the vorticity-bearing ambient flow”, a rather wide (and unexplained) extrapolation of what is understood as an instability. This seems to imply that unstable fluctuations are carried by fluid velocity, which is incorrect for the flow considered in that paper (spirals in Taylor–Couette flows) at finite Reynolds numbers and where the advection of vorticity by the fluid is far from perfect, since such an advection exists in the inviscid limit only. Coles seems to imply that turbulence can grow only as the result of an instability, although I argue below that localised turbulent structures grow by a process of contamination and not by an instability, local or not. Therefore it seems pertinent to reconsider first what is meant by instability in the context of fluid mechanics and in parallel flows.

Stability theory is almost as old as Science as we know it and it kept a strong relationship to fluid mechanics from its very beginning. 2300 years ago Archimedes of Syracuse (Sicily) solved the problem of stability of what we would call 2D floating bodies with a parabolic cross section [3]. Using a geometrical method he proved that, if the floating body is a parabolic cylinder of uniform mass density cut horizontally above a certain height, its vertical equilibrium becomes unstable against tilting. Archimedes even found the new equilibrium positions. This was the beginning of studies of stability in fluid mechanics. As it is out of question to review here the whole history of the field, I jump to another very significant development of this idea of stability in fluid mechanics.

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The next step we shall mention is the explanation of how wind excites waves on the surface of the sea. The theory of waves without wind begins in Newton's Principia, where it was shown that the wave speed is proportional to the square root of the wavelength. This result was established (without solving anything like a partial differential equation) by neglecting the air, windy or not, and by neglecting nonlinear effects, this being correct, as pointed by Newton, whenever the slope of the surface is small. This does not explain the obvious relationship between the wind strength and the wave height. A first link between water waves and wind was established by Kelvin and by Helmholtz almost two centuries after the Principia. In separate works they considered the dynamics of small-amplitude fluctuations of the sea surface under the effect of a wind blowing at uniform speed, all this done in the framework of inviscid fluid dynamics. Although the instability of the fluctuations derived in this way is weak, there is presently no good theory explaining how the linear Kelvin–Helmholtz instability is saturated by dissipation phenomena (mostly by wave breaking [4]).

This theory of Kelvin–Helmholtz instability in the linear approximation made the model of many subsequent studies of linear stability, culminating with the thesis by Werner Heisenberg in 1924 under the guidance of Sommerfeld. In this masterpiece of WKB analysis, Heisenberg [5] showed that, with viscosity included, the plane Poiseuille flow is linearly unstable above a critical Reynolds number, although it is always linearly stable without viscosity. In 1966, Iordanskii and Kulikovskii [6] showed that this flow is convectively, not absolutely, unstable. Although Heisenberg explained his paradoxical result (friction is responsible for instability), it met a strong opposition and various (incorrect) proofs of its "erroneous" character were published.

With the advent of artificial flight and motorcar industry, fluid mechanics became an applied science with many challenges to meet. Therefore the understanding of real flows at moderate to large Reynolds numbers became an urgent matter. Reynolds [1] himself set the stage by studying the experimental transition to turbulence in pipe flows. This led him to introduce what is now called the Reynolds number. He tried to show that the transition occurs at a well defined value of this number, a point hard to make in this case, because the transition is subcritical. As reported below, Reynolds, although he was not by far clear in his statement, seemed to make a distinction between subcritical and supercritical bifurcation. Moreover, he was well aware that fluctuations, if of sufficient amplitude, change the mean structure of the flow, introducing so a feedback between this mean flow (if driven by a constant pressure gradient), its Reynolds number, and the turbulent fluctuations. The Reynolds equation relates the mean velocity, the pressure and the Reynolds stress (which can be seen as the contribution of the turbulent fluctuations to the flux of momentum, another name for the stress—this has been rediscovered several times since in various forms.)

Somehow, Reynolds was first to consider the problem of the sub- or supercritical character of the bifurcation to turbulence in parallel flows. After reporting his experiments of transition to turbulence in a pipe he gave a hint that it could not be the result of a *linear* instability. He asked the question (the last one in a list of six): "Did the eddies make their first appearance as small and then increase gradually with the velocity, or did they come suddenly?"

His (unclear) answer was:

"The bearing of the last query may not be obvious; but, as will appear in the sequel, its importance was such that in spite of satisfactory answers to all the other queries, a negative answer to this in respect of one particular class of motion led to the reconsideration of the supposed cause of instability and eventually to the discovery of the instability caused by fluid friction."

2. From Reynolds to Landau: subcritical vs supercritical bifurcation

After Reynolds' work, many experimental studies put in evidence that parallel flows bifurcate to turbulence via a regime, at intermediate Reynolds number, such that turbulence is localised in well-separated domains having received various names. In careful studies Emmons [7] showed that a Blasius boundary layer shows beyond a range of Reynolds number what are called now "Emmons spots" growing with a well-defined arrowhead shape surrounded by laminar flow and turbulent inside (although with recognisable roll structures of axis in the streamwise direction).

This coexistence of laminar and turbulent domains (turbulent flashes in the words of Reynolds) was unexplained at the time of those observations. Such a coexistence can be stationary, in the Taylor–Couette case, for instance. In a paper presented at a Conference at Los Alamos in 1985 [8], I related this coexistence to the *subcritical* character of the bifurcation.

As this notion of a subcritical bifurcation is going to be central for the developments to come, it should be made more precise. A supercritical instability is an instability growing slowly near a threshold and saturating at a finite amplitude tending to zero as the threshold is reached from above, supposing that above the threshold there is a linear instability and none below. That the instability is sub- or supercritical depends on nonlinear effects. Qualitatively, "subcritical" means that the growth of the amplitude of the fluctuations tends to increase even more their rate of growth. On the contrary fluctuations of finite, even small amplitude have a negative effect on the rate of growth in the case of a "supercritical" bifurcation. However this addition of a nonlinear rate of instability to an already linearly unstable fluctuation does not exhaust all possibilities, because there are examples of flows that remain linearly stable for *all* values of the Reynolds number, like plane Couette flow or pipe Poiseuille flow. Nevertheless this kind of flow is subcritically unstable, like flows becoming linearly unstable at a given Reynolds number, this being the case of the plane Poiseuille flow. They can be considered as subcritical because for Reynolds number above a certain threshold, a turbulent state can exist with steady statistical properties, whereas below this threshold, only the laminar state can maintain itself forever. Somehow, in this case,