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Micromechanics of granular materials – A tribute to Ching S. Chang

Transient rolling friction model for discrete element simulations of sphere assemblies

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ABSTRACT

The rolling resistance between a pair of contacting particles can be modeled with two mechanisms. The first mechanism, already widely addressed in the DEM literature, involves a contact moment between the particles. The second mechanism involves a reduction of the tangential contact force, but without a contact moment. This type of rotational resistance, termed creep-friction, is the subject of the paper. Within the creep-friction literature, the term "creep" does not mean a viscous mechanism, but rather connotes a slight slip that accompanies rolling. Two extremes of particle motions bound the range of creep-friction behaviors: a pure tangential translation is modeled as a Cattaneo–Mindlin interaction, whereas prolonged steady-state rolling corresponds to the traditional wheel–rail problem described by Carter, Poritsky, and others. DEM simulations, however, are dominated by the transient creep-friction rolling conditions that lie between these two extremes. A simplified model is proposed for the three-dimensional transient creep-friction rolling of two spheres. The model is an extension of the work of Dahlberg and Alfredsson, who studied the two-dimensional interactions of disks. The proposed model is applied to two different systems: a pair of spheres and a large dense assembly of spheres. Although creep-friction can reduce the tangential contact force that would otherwise be predicted with Cattaneo–Mindlin theory, a significant force reduction occurs only when the rate of rolling is much greater than the rate of translational sliding and only after a sustained period of rolling. When applied to the deviatoric loading of an assembly of spheres, the proposed creep-friction model has minimal effect on macroscopic strength or stiffness. At the micro-scale of individual contacts, creep-friction does have a modest influence on the incremental contact behavior, although the aggregate effect on the assembly's behavior is minimal.

1. Introduction

In a Discrete Element (DEM) simulation of a granular material, each particle is represented as a discrete object that interacts with neighboring particles at its contacts. Each contact interaction yields the contact force that results from the movements of the two particles. Some recent DEM codes also include a possible *contact moment* that arises from the pair-wise rotational interactions of the particles. With this form of *rotational resistance*, the contact interaction is stiffened

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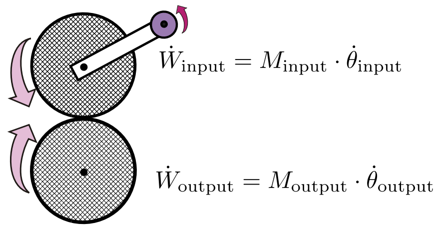

Fig. 1. Input and output power, with $M_{\text{input}} \cdot \dot{\theta}_{\text{input}} \geqslant M_{\text{output}} \cdot \dot{\theta}_{\text{output}}$.

between the particle pairs, and the entire assembly is hardened and strengthened as a result. Besides imparting a macro-scale hardening, Iwashita and Oda [1] showed that contact moments also alter the internal length scale of an assembly, an effect that is evidenced by a change in the thickness and character of localization features such as shear bands. Ai et al. [2] provide a comprehensive survey of the many mechanisms that have been proposed for implementing a contact moment in DEM codes, mechanisms that include various combinations of rotational springs and rotational dissipation elements, such as rotational dampers and sliders. In the Paper, we consider an alternative form of rolling resistance and dissipation that does not involve contact moments. This mechanism, termed *creep-friction*, has its origins in the rolling friction literature and results from micro-slip at particle contacts. Within the creep-friction literature, the term "creep" does not mean a viscous mechanism, but rather connotes a gradual slip during rolling. Creep-friction has, as yet, received little attention within the DEM community. We begin by clarifying the distinction between two general forms of rotational resistance: contact moments and creep-friction. We then focus on the creep-friction mechanism of rotational resistance and describe its similarities and differences with translational resistance, in particular that of Cattaneo [3] and Mindlin [4], a type of sliding resistance that is widely used in DEM codes. Although the creep-friction mechanism has received much interest over the past century, no exact solutions have been offered for many basic problems in two-dimensional interaction, let alone for the general three-dimensional conditions that one encounters in granular flows. The paper proposes an approximate model for creep-friction (Section 2) and then uses this model to gauge the relative importance of creep-friction in common granular phenomena (Section 3).

1.1. Two categories of rolling resistance

Two categories of rolling resistance can be distinguished by considering a pair of smooth cylindrical rollers of equal radius, one being driven by the other (Fig. 1). Various mechanisms can lead to energy dissipation, causing the input power $M_{\text{input}} \cdot \dot{\theta}_{\text{input}}$ to exceed the output power $M_{\text{output}} \cdot \dot{\theta}_{\text{output}}$ (here, M and $\dot{\theta}$ represent moments and rotational velocities). With the first category of rolling resistance, a contact moment acts between the two rollers so that the input moment exceeds the output moment, yet preserving the rotational velocities. This type of rolling resistance was used in the DEM simulations of Iwashita and Oda [1] and is now available in many DEM codes (see Ai et al. [2] for a survey). The presence of a contact moment is most obvious in the interaction of two gears that rotate with synchronous velocities, but in which the teeth rub in a manner that causes a torque reduction between the two gears: a reduction that can be modeled as a contact moment. A contact moment is also produced when the two contacting materials are inelastic, which results in an asymmetry of stress within the small contact area between the two rolling objects. As rolling proceeds, the leading portion of the contact is continually loaded while the trailing portion is unloaded, and any difference in the loading and unloading stiffnesses (as a result of plasticity, viscosity, crushing, or sticky adhesion) will produce a small contact moment.

The second category of rolling resistance, also illustrated in Fig. 1, is the focus of the Paper. With this mechanism, the absence of a contact moment preserves the torque (that is, $|M_{\text{input}}| = |M_{\text{output}}|$), but the output rotational velocity is diminished between the driving roller and the driven roller. This latter type of resistance, termed creep-friction, results from micro-slip between the two rollers within their small contact region, causing energy dissipation in the absence of a contact moment. Both Reynolds [5] and Johnson [6] likened this type of micro-slip dissipation to the slip that occurs between a belt and its pulley. Creep-friction is the dominant dissipation mechanism in most rolling stock and, hence, received early attention in the investigation of rail–wheel interactions. As a train engine rolls steadily upon its rails, the wheel rim moves slightly faster than the train itself, and this small disparity increases as the grade steepens until, in the limit, the train stalls as the wheels spin futilely upon their rails. The paper concerns this form of rolling resistance and dissipation.

1.2. Creep-friction vs. Cattaneo–Mindlin sliding

Although both result from micro-slip within the contact zone, a distinction must be made between the mechanics of creep-friction rolling and that of translational sliding (see Fig. 2 for a summary of these differences). The problem of translational interaction between two elastic spheres was independently solved by Cattaneo [3] and Mindlin [4], and this purely translational problem will be referred to as the *Cattaneo–Mindlin problem*. Its solution is in the form of a relationship between the translational (tangential) displacement and the tangential traction and force. This relationship is known to be complexly path-dependent, such that the tangential force depends upon the previous sequence of both tangential and normal displacements. Mindlin and Deresiewicz [7] developed solutions for eleven sequences of such loading and unloading,