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Micromechanics of granular materials – A tribute to Ching S. Chang

Particle size and boundary geometry effects on the bulk friction coefficient of sheared granular materials in the inertial regime

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ABSTRACT

Glass beads of varying diameters ($d = 2, 3, 4$, and 5 mm) are used to measure the ratio of shear-to-normal stress, or bulk friction coefficient, generated inside an annular shear cell at high shearing rates. The effects of the particle size, the solids concentration, and the shear rate are explored. It is found that (1) for a given particle size, the magnitude of the bulk friction coefficient decreases with increasing solids concentration, (2) for a given solids concentration, the bulk friction coefficient decreases with increasing particle size, and (3) the bulk friction coefficient is independent of the shear rate except for cases with low solids concentration, where it decreases with increasing shear rate. The boundary geometry is found to affect bulk friction only for dilute (low solids concentration) flows involving small particles.

1. Introduction

In geotechnical engineering, the shear strength of a soil is measured using the angle of internal friction. In the absence of cohesion, the tangent of this angle, also known as the bulk friction coefficient of the soil, is equal to the ratio of shear-to-normal stress at the plane of failure. This is usually the failure criterion used in engineering design. A similar idea was adopted by Bagnold [1] in his work in sediment transport. In such flows the major momentum transfer is through particle collisions, referred to as the "inertial" regime. Bagnold assumed that a dynamic yield criterion is satisfied at the boundary between movable and immovable grains. He based this on measurements of shear and normal stresses generated by shearing a neutrally buoyant dispersion of spheres between two concentric cylinders [2]. In his experiments the bulk friction coefficient varied from 0.32 to 0.75 depending upon the flow conditions and characteristics of the granular material. This yield criterion for granular fluid flows was later validated by Hanes and Inman [3,4] using an annular shear cell similar to the one employed in the current study. During annular shear cell experiments using glass beads of 1.1 and 1.85 mm diameter they noted that in many cases the flow was divided into two distinct regions: (1) a shearing (upper) region and (2) a non-shearing (lower) region. Their data showed a nearly constant (with respect to shear rate) bulk friction coefficient at the boundary between a rapidly flowing granular layer and a stationary but movable granular bed. Their data shows that

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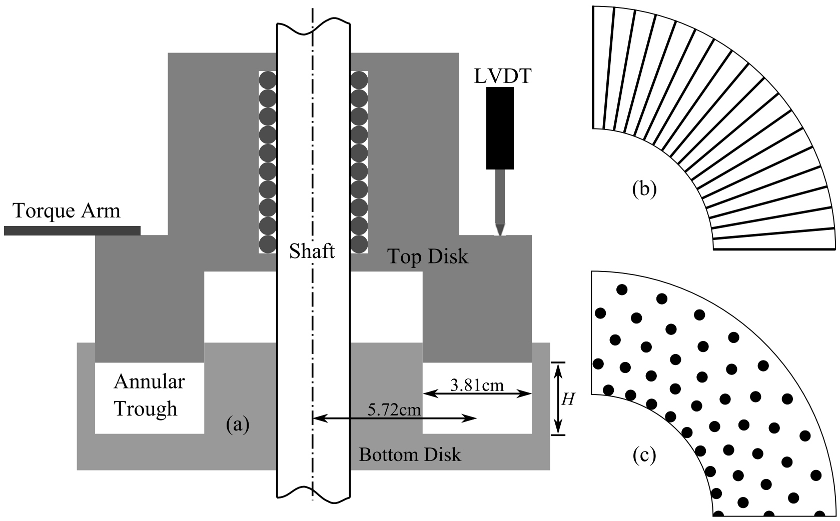

Fig. 1. Schematic of (a) the annular shear cell used in the experiments, (b) boundary type 1, and (c) boundary type 2.

the bulk friction coefficient is nearly independent of particle diameter, for that narrow range in size. More recently Orlando and Shen [5] showed that the particle size and boundary geometry have a marked effect on the shear stress measured on the upper boundary of an annular shear cell at high shearing rates. However, the effects of particle size and boundary geometry on the bulk friction coefficient of a granular material were not explored. In the current work their data is analyzed in terms of the bulk friction coefficient and the effects that parameters such as particle size, solids concentration, shear rate, and boundary geometry have on it.

2. Experiments and results

Orlando and Shen [5] describe annular shear cell experiments used to study the effect of particle size and boundary geometry on the measured shear stress at the upper boundary. The upper and lower boundaries were roughened by machining either 72 equally distributed half-cylinders (boundary type 1) of 1.59 mm diameter along the radial direction or hemispheres (boundary type 2) of 3.18 mm diameter in a polar hexagonal close packed configuration. The materials used in their work were glass beads of nominal diameter $d = 2, 3, 4$, and 5 mm with a density of 2.6 g/cm^3. In the current study their data is presented in terms of the bulk friction coefficient measured at the upper boundary. A schematic of the shear cell used in the experiments and the boundary geometries are shown in Fig. 1. The annular trough was machined out of aluminum into the bottom circular disk and was driven by a 3/4 hp DC servo-controlled motor. The top disk was mounted on the shaft using bearings to allow smooth vertical displacement and rotation, and alongside a set of counterweights was used to adjust the normal load on the granular material (0–4.5 kPa). The torque developed by the flow was measured using a full bridge thin beam load cell attached to the torque arm, and the displacement of the top disk was measured using a Linear Variable Differential Transformer (LVDT). Data from the LVDT and the load cell were recorded simultaneously. The angular velocity of the bottom disk was measured using a tachometer.

The experiments were run by depositing a known mass of glass beads in the annular trough. The top disk was allowed to rest fully on the granular bed, and the initial height of the bed was measured using Vernier calipers. After the normal load was adjusted to a desired value by using counterweights, the motor was started and the speed of the annular trough increased until the desired height H in Fig. 1, and thereby the desired average solids concentration ν was reached. After readings of the torque and rotational speed were taken, the normal load was increased. The rotational speed was also increased until the same height H was obtained, and new readings of the torque and rotational speed were taken for the current normal load. This was done until enough data points were collected. The shear and normal stresses generated by the flow at the upper boundary and the apparent shear rate were calculated as in Savage and Sayed [6]:

$$\tau = \frac{3}{2}\frac{T}{\pi(R_o^3 - R_i^3)} \tag{1}$$

$$\sigma = \frac{N}{\pi(R_o^2 - R_i^2)} \tag{2}$$

$$\gamma = \frac{\omega(R_o + R_i)}{2H} \tag{3}$$

where τ, σ and γ are respectively the shear and normal stress at the upper boundary and the apparent shear rate. T is the measured torque, N is the applied normal load, ω is the angular velocity of the annular trough, and R_o and R_i are the