C. R. Mecanique 341 (2013) 701–708

Contents lists available at SciVerse ScienceDirect

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Refined theory and decomposed theorem of transversely isotropic thermoporoelastic beam

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ARTICLE INFO

Article history:
Received 4 April 2013
Accepted 15 July 2013
Available online 12 August 2013

Keywords:
Thermoporoelastic
Beam
Transversely isotropic
Refined theory
Decomposed theorem
General solution

ABSTRACT

The refined theory of transversely isotropic beam is analyzed. Based on the transversely isotropic thermoporoelastic theory, a refined theory for bending beam is derived using the general solution and the Lur'e method without ad hoc assumptions. First, the expressions for all of the displacements and stress components of a transversely isotropic thermoporoelastic beam were obtained in terms of four functions with one independent variable. Second, using homogeneous boundary condition, the refined equation and the decomposed form of the thermoporoelastic beam were obtained. Finally, the approximate equations and solutions for the beam under general anti-symmetric loadings were derived from the refined theory.

1. Introduction

The consolidation theory of porosity media was extended from one to three dimensions by Biot [1], and the consolidation theory was improved more perfectly. Huang [2] and others gave the analytical solution of pore water pressure, stress and displacement of the two-dimensional consolidation problem [3–6].

Without ad hoc assumptions, Cheng [7] developed the refined theory for bending of isotropic plates directly from the three-dimensional theory of elasticity by using the solution of the plates and Lur'e method [8]. Under homogeneous boundary conditions, the refined theory of plate is exact and consists of three parts: the bi-harmonic equation, the shear equation and the transcendental equation. A parallel development on the plate theory was constructed by Gregory. In 1992, Gregory [9] provided a rigorous proof of the decomposed form of isotropic plates. The two-plate theory has been extended to the study of various material boards, such as transversely isotropic [10], thermoelastic [11], magnetoelastic [12], and piezoelectric plates [13]. In 2005, the connection between the refined theory and the decomposed theorem of an isotropic elastic plate was analyzed by Zhao and Wang [14]. The equivalence of the refined theory and the decomposed theorem of an isotropic plate were obtained. In 2007, the refined theory of thermoelastic rectangular plates [15] and thermoelastic plane problems [16] were obtained by Gao and Zhao.

In this paper, the research into the refined theory is extended to the study of the transversely isotropic thermoporoelastic beam. In the next section, the basic equations and notations are stated. In Section 3, the decomposed theorem under homogeneous boundary conditions is studied. The decomposed form is consistent with the interior state, the transcendental state, the pore pressure state and the thermal state. In Sections 4, 5, the approximate equations and the solutions for the beam under general anti-symmetric loadings are derived directly from the refined theory.

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1631-0721/$ – see front matter
http://dx.doi.org/10.1016/j.crme.2013.07.001

2. Equations and notations

A transversely isotropic thermoporoelastic beam occupies the region:

$$\Omega = \{(x,z) \mid x \in D,\ |z| \leqslant t\} \tag{1}$$

where D is the cross-section of the beam, which has thickness $2t$, the z-axis being perpendicular to the isotropic plane of the medium in a Cartesian system (x,z). The constitutive equations for the transversely isotropic body in the two-dimensional linear elasticity are described to be:

$$\sigma_{xx} = C_{11}\frac{\partial u}{\partial x} + C_{13}\frac{\partial w}{\partial z} - \alpha_1 P - \beta_1 T, \qquad \sigma_{zz} = C_{13}\frac{\partial u}{\partial x} + C_{33}\frac{\partial w}{\partial z} - \alpha_3 P - \beta_3 T$$
$$\sigma_{zx} = C_{44}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right), \qquad P = M\left(\xi - \alpha_1\frac{\partial u}{\partial x} - \alpha_3\frac{\partial w}{\partial z} + \beta_m T\right) \tag{2}$$

where σ_{xx}, σ_{zz} are the normal stresses, σ_{zx} is the shear stress, and u and w are displacements in the respective Cartesian directions, P and T are changes in the pore pressure and temperature. ξ is the variation of the fluid content. C_{ij}, α_1 (α_3, M) and β_1 (β_3, β_m) are the elastic moduli, Biot's effective stress coefficients and thermal constants. It is noted that C_{ij}, α_i, β_i can be expressed in terms of engineering contents such as Young's moduli, Poisson's ratio, etc.

The general solution of thermoporoelastic beam has the following expression [17]:

$$u = -\sum_{i=1}^{4}\frac{\partial\psi_i}{\partial x}, \qquad w = \sum_{i=1}^{4}\mu_{i1}\frac{\partial\psi_i}{s_i\partial z}, \qquad P = \mu_{32}\frac{\partial^2\psi_3}{s_3^2\partial z^2}, \qquad T = \mu_{43}\frac{\partial^2\psi_4}{s_4^2\partial z^2} \tag{3}$$

where:

$$\mu_{i1} = s_i\frac{a_2 s_i^4 - b_2 s_i^2 + c_2}{a_1 s_i^4 - b_1 s_i^2 + c_1} \quad (i = 1,2,3,4), \qquad \mu_{12} = \mu_{22} = \mu_{42} = \mu_{13} = \mu_{23} = \mu_{33} = 0$$
$$\mu_{32} = \frac{(a_0 s_3^4 - b_0 s_3^2 + c_0)(\lambda_{33}s_3^2 - \lambda_{11})}{a_1 s_3^4 - b_1 s_3^2 + c_1}, \qquad \mu_{43} = \frac{(a_0 s_4^4 - b_0 s_4^2 + c_0)(\kappa_{33}s_4^2 - \kappa_{11})}{a_1 s_4^4 - b_1 s_4^2 + c_1} \tag{4}$$

$$a_0 = C_{33}C_{44}, \qquad b_0 = C_{11}C_{33} - C_{13}^2 - 2C_{13}C_{44}, \qquad c_0 = C_{11}C_{44}$$
$$a_1 = (C_{13} + C_{44})(\kappa_{33}\beta_3 + \lambda_{33}\alpha_3) - C_{33}(\kappa_{33}\beta_1 + \lambda_{33}\alpha_1)$$
$$b_1 = (C_{13} + C_{44})(\kappa_{11}\beta_3 + \lambda_{11}\alpha_3) - C_{44}(\kappa_{33}\beta_1 + \lambda_{33}\alpha_1) - C_{33}(\kappa_{11}\beta_1 + \lambda_{11}\alpha_1)$$
$$c_1 = -C_{44}(\kappa_{11}\beta_1 + \lambda_{11}\alpha_1), \qquad a_2 = C_{44}(\kappa_{33}\beta_3 + \lambda_{33}\alpha_3)$$
$$b_2 = C_{11}(\kappa_{33}\beta_3 + \lambda_{33}\alpha_3) + C_{44}(\kappa_{11}\beta_3 + \lambda_{11}\alpha_3) - (C_{13} + C_{44})(\kappa_{33}\beta_1 + \lambda_{33}\alpha_1)$$
$$c_2 = C_{11}(\kappa_{11}\beta_3 + \lambda_{11}\alpha_3) - (C_{13} + C_{44})(\kappa_{11}\beta_1 + \lambda_{11}\alpha_1) \tag{5}$$

in which κ_{11} (κ_{33}) and λ_{11} (λ_{33}) are the coefficients of permeability and the thermal conductivity, and where ψ_i ($i = 1,2,3,4$) are the harmonic functions that satisfy the following equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{s_i^2\partial z^2}\right)\psi_i = 0 \quad (i = 1,2,3,4) \tag{6}$$

where $s_3^2 = \kappa_{11}/\kappa_{33}$, $s_4^2 = \lambda_{11}/\lambda_{33}$, s_1^2 and s_2^2 are two roots of the following equation (set $s_1^2 \neq s_2^2$):

$$a_0 s^4 - b_0 s^2 + c_0 = 0 \tag{7}$$

Lekhnitskii [18] proved that the numbers s_1 and s_2 for any transversely isotropic body can be real or complex (with a real part different from zero), but cannot be purely imaginary.

Since the stresses in the bending beam are anti-symmetrical about mid-plane $z = 0$, this induces that u and v are the odd function about z, and w is the even function about z. Using the Lur'e method [8], we have the following solutions of (7):

$$\psi_i = \frac{\sin(zs_i\partial_x)}{s_i\partial_x}g_i(x) \quad (i = 1,2,3,4) \tag{8}$$

in which g_i ($i = 1,2,3,4$) are unknown functions of x, yet to be determined, and $\partial_x = \partial/\partial x$, and: