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Out of Equilibrium Dynamics

Detonation-propelled shocks in tubular charges as a two-phase problem

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ABSTRACT

The article is concerned with the experimentally known phenomenon of the precursor shock driven by the detonation of a tubular charge. It is shown that the basic aspects of the effect may be successfully captured within a one-dimensional two-phase version of the Chapman–Jouguet (CJ) theory. A modified CJ principle for determination of the detonation and precursor shock velocities is discussed.

1. Introduction

Zeldovich [1], in his renowned paper of 1940 on the ZND model, asked a curious question: Can the gaseous detonation propel a precursor shock, as occurs in deflagrative combustion? This issue seems never to have enjoyed further discussion, presumably due to the realization that for the self-sustained CJ detonation higher speed precursor shocks are untenable (see Appendix A). It transpires, however, that for multiphase systems Zeldovich's question may well have a positive answer.

In 1947 Woodhead [2] reported an observation that detonation running through a solid charge with a longitudinal air-filled channel is accompanied by an air-shock propelled ahead of the detonation at a velocity around twice that of the detonation, Fig. 1. The emerging post-shock temperature is around 10,000 K, causing an intense luminosity of the shocked gas [2–5]. The gas flow developing in the channel is highly energetic (~ 10 km/s), which may be of practical interest, e.g. for launching small high-velocity projectiles [6,7] – an effective means for impact engineering [8].

Although there is now a substantial volume of literature on the subject [2–19], a first-principle understanding of the key interactions controlling the channel effect is still incomplete.

In the present study the problem is approached from the viewpoint that the tubular charge may be considered as a special case of a gas-permeable porous explosive. One therefore may expect that the channel effect should have a counterpart in the one-dimensional two-phase (gas–solid) picture widely employed in modeling of porous energetic materials. Indeed, as shown below the channel effect is quite generic to gas-permeable systems, and may be successfully captured even within the zero-reaction-zone CJ model. The familiar CJ principle of the minimum mass flux through the detonation, combined with the requirement of the maximum mass flux through the precursor shock, allows fixing both the detonation velocity as well as the velocity of the shock.

2. Formulation

Discarding transport effects, the set of conservation equations for the gas–solid mixture reads [20],

mass balance,

$$\frac{\partial}{\partial t}(\varphi_g\rho_g+\varphi_s\rho_s)+\frac{\partial}{\partial x}(\varphi_g\rho_g u_g+\varphi_s\rho_s u_s)=0 \tag{1}$$

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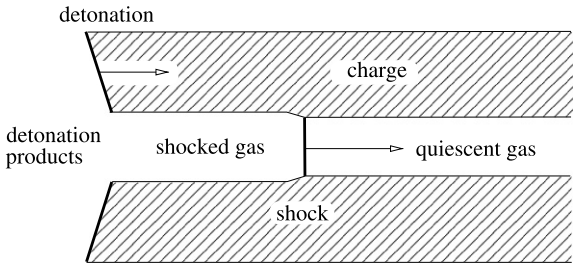

Fig. 1. Diagram for the channel effect. Arrows (also in Fig. 2) represent detonation and precursor shock velocities [5].

momentum balance,

$$\frac{\partial}{\partial t}(\varphi_g\rho_g u_g+\varphi_s\rho_s u_s)+\frac{\partial}{\partial x}\left(\varphi_g\rho_g u_g^2+\varphi_g P_g+\varphi_s\rho_s u_s^2+\varphi_s P_s\right)=0 \tag{2}$$

energy balance,

$$\frac{\partial}{\partial t}(\varphi_g\rho_g E_g+\varphi_s\rho_s E_s)+\frac{\partial}{\partial x}\left[\varphi_g u_g(\rho_g E_g+P_g)+\varphi_s u_s(\rho_s E_s+P_s)\right]=0 \tag{3}$$

$$E_g=e_g+\frac{1}{2}u_g^2,\qquad E_s=e_s+\frac{1}{2}u_s^2 \tag{4}$$

$$\varphi_g+\varphi_s=1 \tag{5}$$

Here the subscripts s and g refer to the solid and the gas phase, respectively. The state variables for the phases are: φ_g, φ_s – volume fractions; P_g, P_s – pressures; T_g, T_s – temperatures; ρ_g, ρ_s – densities; u_g, u_s – flow velocities; e_g, e_s – specific internal energies; E_g, E_s – total specific energies.

As in the CJ model, the reaction, converting the solid explosive into gaseous products, is assumed to occur instantly, i.e. detonation is treated as a reactive discontinuity. Hence, $\varphi_g=\varphi_0$, $\varphi_s=1-\varphi_0$ ahead of the detonation front, and $\varphi_g=1$, $\varphi_s=0$ in the products (φ_0 is prescribed).

To complete the formulation the following additional assumptions are made:

1. The solid phase is incompressible and static,

$$\rho_s=\rho_{s0},\qquad u_s=0 \tag{6}$$

2. Pressures in gas and solid phases are identical,

$$P_g=P_s=P \tag{7}$$

which is a common enough premise in modeling two-phase media [21]. By virtue of conditions (6), this assumption does not violate the hyperbolic character of the system.

3. Temperatures T_g and T_s are coupled through the relation,

$$T_s=T_0+k(T_g-T_0) \tag{8}$$

where T_0 is the ambient temperature, and k is the heat-exchange parameter. At $k=0$ the phases are thermally separated, while at $k=1$ the phases are in thermal equilibrium – an unlikely situation in supersonic combustion.

4. Equations of state for gas and solid phases are specified as,

$$e_g=c_g T_g,\qquad e_s=c_s T_s+Q \tag{9}$$

$$P=\left[\gamma(\varphi_g)-1\right]\rho_g e_g,\qquad \rho_s=\rho_{s0} \tag{10}$$

Here Q is the heat-release; c_g, c_s – specific heats, assumed to be constant; $\gamma(\varphi_g)$ – adiabatic index; $\gamma_p=\gamma(1)$, representing strongly pressurized detonation products, is about twice $\gamma_c=\gamma(\varphi_0)$ for the gas inside the channel.

5. To somewhat reduce mathematical clutter in the subsequent discussion it is set,

$$c_s=\gamma_c c_g\quad(=c_p) \tag{11}$$

For solid propellants such as cyclic nitramines RDX and HMX, c_s is slightly above c_p [22]. So the case of $c_s=c_p$ seems to be quite representative [23].

Ahead of the precursor shock,

$$P=P_0,\qquad \rho_g=\rho_{g0},\qquad T_g=T_0,\qquad u_g=u_{g0}=0,\qquad \varphi_g=\varphi_0 \tag{12}$$

Upon initiation and some transient time period the profiles of pressure, temperature, density, and flow-velocity are expected to assume simple step-like (though unsteady) structures as sketched in Fig. 2.