C. R. Mecanique 341 (2013) 727–742

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

www.sciencedirect.com

Simulation of the shear strength for unsaturated soils

Hiram Arroyo [a,*], Eduardo Rojas [a], María de la Luz Pérez-Rea [a], Jaime Horta [a], José Arroyo [b]

[a] Faculty of Engineering, Universidad Autónoma de Querétaro, Centro Universitario, Cerro de las Campanas, 76160, Querétaro, Qro., Mexico
[b] Faculty of Civil Engineering, Universidad Michoacana de San Nicolás de Hidalgo, Centro Universitario, Avenida Francisco J. Mújica S/N, 58030, Morelia, Mich., Mexico

ARTICLE INFO

Article history:
Received 4 July 2013
Accepted 11 October 2013
Available online 1 November 2013

Keywords:
Unsaturated soils
Shear strength
Modeling
Effective stresses
Geotechnical models
Retention curve

ABSTRACT

The simulation of the hydro-mechanical behavior of unsaturated soils is becoming a subject of major importance in soil mechanics. However, unlike the laws governing the behavior of saturated soils, those used to describe the behavior of unsaturated soils still lack of simplicity for common engineering practice. This is why it is important to reconcile saturated and unsaturated soil mechanics and establish a unified theory. In this paper, we use the same strength equation of saturated soils in unsaturated materials and verify that a single failure surface is obtained for any value of suction in wetting and drying paths.

1. Introduction

Shear strength is one of the most important engineering properties of soils. This is because most civil engineering works require the knowledge of the soil resistance to design safe structures.

The shear strength of soils can be measured in equipments that apply three-dimensional pressures (directions 1, 2 and 3). In this case, the triaxial cell or the true triaxial apparatus can be used. When different pressures are applied in two or three directions, they cause shear stresses that are supported by the internal structure of soils. When these acting stresses exceed the strength of the material, it is said that the soil fails. In the case of saturated materials, this failure state is identified through the final deviator stress ($q_{\mathrm{f}} = \sigma_1 - \sigma_3$) and the final mean effective stress ($p' = (\sigma_1' + 2\sigma_3')/3$) where the effective stress (σ') is defined as the difference between the total stress and the pore pressure. If we plot these values ($p_{\mathrm{f}}', q_{\mathrm{f}}$) under different confining stresses for a saturated soil, they exhibit a single failure line with a slope called M. In this sense, the failure deviator stress is equal to Mp_{f}' and represents the maximum internal resistance in a failure plane when the soil sample is subjected to external loads:

$$q_{\mathrm{f}} = Mp_{\mathrm{f}}' \tag{1}$$

In contrast, the shear strength in unsaturated soils can be predicted using two different approaches. The first one is based on the independent stress state variables and considers that the strength due to mechanical loading is independent of the strength due to suction [1]. The variables used to establish the governing equations in this approach are the mean net stress $p_{\mathrm{net}} = (\sigma_1 + 2\sigma_3)/3$, the deviator stress $q = \sigma_1 - \sigma_3$ and suction s. Matric suction $s = u_{\mathrm{a}} - u_{\mathrm{w}}$ represents the pressure difference of the fluids in the soil pores: air (u_{a}) and water (u_{w}). Suction is included as a third variable in order to take

* Corresponding author. Tel.: +52 442 192 1200x6031; fax: +52 442 192 1200x6005.
E-mail address: hiramarroyo@hotmail.com (H. Arroyo).

1631-0721/$ – see front matter
http://dx.doi.org/10.1016/j.crme.2013.10.005

into account correctly the effect of moisture on the strength of the material. This approach considers the stress state as a function of these three variables $f = f(p_{\mathrm{net}}, q, s)$.

The second approach is similar to the saturated case, where the strength of the material is linked to the shear stress and the mean effective stress. Models written in terms of effective stress couple the mechanical and hydraulic behavior of the material in this single variable. For the case of unsaturated soils, the most popular effective stress relationship is Bishop's effective stress equation, which in tensorial form is written as:

$$\sigma'_{ij} = \sigma^t_{ij} - u_a\delta_{ij} + \chi s\delta_{ij} \tag{2}$$

where σ^t_{ij} represents the total stress applied at the boundaries of the soil, δ_{ij} is the Kronecker delta and χ is Bishop's parameter, closely related to the degree of saturation of the soil [2]. More recent studies relate this parameter to saturated, dry, and unsaturated fractions of soil [3].

When relationship (2) is multiplied by the Kronecker delta, the mean effective stress is expressed in terms of the mean net stress (p_{net}) and suction:

$$p' = p_{\mathrm{net}} + \chi s \tag{3}$$

Several researchers have made attempts to establish analytical equations to obtain parameter χ. The most simplistic of them assumes that it is equal to the degree of saturation of the material ($\chi = S_r$).

This paper focuses on the prediction of the shear strength of unsaturated soils using the effective stress concept as it shows some advantages when compared to the independent stress state variables approach. Among other advantages, effective stress models require less constitutive parameters to be formulated. Furthermore, in order to calibrate these parameters, more simplistic and economical laboratory tests are required [4]. In this paper, we only consider the effect of matric suction and neglect those produced by other types of suction.

A model to predict the strength of unsaturated soils is proposed herein. First, an analytical equation to determine Bishop's parameter based on the analysis of the equilibrium of the soil phases (solid, liquid, gas) is presented. Then it is demonstrated that it can be applied to any type of soil. Later, a probabilistic porous model is developed to generate the parameters required to obtain the effective stresses in unsaturated materials. Additionally, proper details regarding the calculation of the variables that intervene in the effective stress equation proposed herein are provided. Finally, some numerical and experimental results comparisons are presented.

2. Bishop's parameter determination

Fig. 1(a) outlines a porous structure composed of solids and pores. These pores can be of two different types: sites and bonds. The sites contain the largest part of the voids and can be subdivided in macropores and mesopores. The bonds or throats are the elements that interconnect the sites.

Consider that a soil is initially subjected to a high suction and all pores are dry. If suction is reduced by steps, at certain point, water intrudes into the system. Initially, only the smaller bonds located at the boundaries of the sample saturate. With further reductions in suction, other larger elements start to saturate. Fig. 1(b) shows the distribution of water in the system at a certain stage of the wetting process. It can be observed that the system is now composed of pores that may be either dry or saturated (both sites and bonds). Additionally, it can be observed that solids can be completely surrounded by either saturated or dry pores, or even they can be surrounded by a combination of them. For example, solid 6 in Fig. 1(b) is completely surrounded by dry pores and in that sense it is called a dry solid. Instead, solids 1 and 3 are completely surrounded by saturated pores and therefore they are called saturated solids. Finally, solids 2, 4 and 5 are surrounded by a combination of saturated and dry pores, then they are called unsaturated solids. Notice that the definitions of the different fractions involve both solids and pores and not the pores alone. The total volume of solids added by the volume of their surrounding pores forms the volume of the fraction, be it saturated, dry, or unsaturated.

It will be seen later that the value of Bishop's parameter as proposed herein depends on the quantification of the different fractions. Because the saturated and the dry fractions do not share common elements, these two fractions are the first ones to be quantified. Then, the unsaturated fraction is quantified from the difference with the total volume of bonds, sites and solids. In this form we ensure that neither element is counted twice.

Let us define the saturated fraction f^{s} as the ratio between the saturated volume (saturated solids and its surrounding pores) and the total volume of the sample V; the saturated (f^{u}) and the dry fraction (f^{d}) are defined in the same way: the unsaturated volume divided by the total volume and, the dry volume divided by the total volume, respectively.

$$f^{\mathrm{s}} = \left(V^{\mathrm{s}}_{\mathrm{SOL}} + V^{\mathrm{s}}_{\mathrm{S}} + V^{\mathrm{s}}_{\mathrm{B}}\right)/V \tag{4a}$$

$$f^{\mathrm{d}} = \left(V^{\mathrm{d}}_{\mathrm{SOL}} + V^{\mathrm{d}}_{\mathrm{S}} + V^{\mathrm{d}}_{\mathrm{B}}\right)/V \tag{4b}$$

$$f^{\mathrm{u}} = \left(V^{\mathrm{u}}_{\mathrm{SOL}} + V^{\mathrm{u}}_{\mathrm{S}} + V^{\mathrm{u}}_{\mathrm{B}}\right)/V \tag{4c}$$

In this way, the condition $f^{\mathrm{s}} + f^{\mathrm{d}} + f^{\mathrm{u}} = 1$ must be satisfied at any value of the degree of saturation. Variables in Eq. (4) show subscripts that refer to either solids (SOL), sites (S), or bonds (B). On the other hand, superscripts indicate whether