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Bounds on the hydrostatic plastic strength of voided polycrystals and implications for linear-comparison homogenization techniques

Bornes de la résistance plastique hydrostatique des polycrystaux poreux et leurs implications sur des techniques basées sur des milieux linéaires de comparaison

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ABSTRACT

A linear-comparison homogenization technique and its relaxed version are used to compute bounds of the Hashin–Shtrikman and the self-consistent types for the hydrostatic strength of ideally plastic voided polycrystals. Closed-form analytical results are derived for isotropic aggregates of various cubic symmetries (fcc, bcc, ionic). The impact of the variational relaxation on the bounds is found to be significantly larger than that previously observed in fully dense polycrystals. So much so that, quite surprisingly, relaxed self-consistent bounds are found to be weaker than non-relaxed Hashin–Shtrikman bounds in some of the material systems considered.

RÉSUMÉ

Une technique d'homogénéisation non linéaire et sa version relaxée sont utilisées pour calculer des bornes de types Hashin–Shtrikman et autocohérent pour la resistance hydrostatique de polycristaux poreux parfaitement plastiques. On en dérive des résultats analytiques pour des agrégats isotropes de différentes symétries cubiques (cfc, ccc, ionique). L'impact sur les bornes de la relaxation variationnelle se révèle être beaucoup plus important que celui précédemment observé dans le cas de polycristaux denses, tant et si bien que des bornes relaxées de type autocohérent s'avèrent être plus faibles que des bornes non relaxées de type Hashin–Shtrikman dans certains systèmes matériels considérés.

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1. Introduction

Several homogenization techniques are already available to bound the plastic strength of polycrystalline solids. The simplest bounds are the upper bound of Taylor [1] and the lower bound of Reuss [2], which depend on one-point microstructural statistics only. Sharper bounds incorporating higher-order statistics were first derived by Dendievel et al. [3] and deBotton and Ponte Castañeda [4] making use of the idea of a linear-comparison medium that is optimally selected via suitably designed variational principles. In particular, the technique of deBotton and Ponte Castañeda allows the use of any linear homogenization approach, such as the Hashin–Shtrikman or the self-consistent approach, to generate the corresponding results for nonlinear plastic polycrystals. Application of these linear-comparison techniques to various classes of fully dense polycrystalline solids have been pursued by Willis [5], Nebozhyn et al. [6,7], Liu et al. [8], and Liu and Ponte Castañeda [9]. In all cases, the nonlinear Hashin–Shtrikman bounds are very close to the Taylor bound, but the nonlinear self-consistent bounds can be quite sharper, especially for low-symmetry solids. This is in part due to the fact that the Hashin–Shtrikman results bound the entire class of polycrystals with prescribed one- and two-point statistics, while the self-consistent results bound the subclass of polycrystals that realize the linear self-consistent estimate.

Idiart and Ponte Castañeda [10,11] later showed that the linear-comparison technique of deBotton and Ponte Castañeda [4] makes implicit use of a relaxation in the variational scheme that weakens the resulting bounds. Eliminating this relaxation leads to sharper bounds at the expense of increasing the computational complexity. The impact of the relaxation in the context of various cubic and hexagonal systems has been recently assessed by Idiart [12]. Modest differences between relaxed and non-relaxed bounds were observed, with the largest amounts corresponding to materials with deficient slip systems. Given that these are materials with a strong heterogeneity contrast, the question arises as to whether larger differences will appear in the context of (two-phase) polycrystalline voided solids where the heterogeneity contrast is infinitely strong. This Note reports linear-comparison bounds of the Hashin–Shtrikman and self-consistent types for the plastic strength of polycrystalline voided solids exhibiting overall isotropic symmetry and subjected to purely hydrostatic loadings. These conditions are of particular theoretical interest since they usually exacerbate differences between theories and, furthermore, allow analytical treatment. It turns out that the impact of the relaxation on the bounds can be significantly larger than that observed in fully dense polycrystals. So much so that, quite surprisingly, relaxed self-consistent bounds are found to be weaker than non-relaxed Hashin–Shtrikman bounds for some of the material systems considered. We conclude the Note by discussing some implications for the use of linear-comparison techniques in the context of voided polycrystals.

2. The polycrystalline solid model

Polycrystals are taken here as random aggregates of perfectly bonded single crystals (i.e., grains) and voids. Following Lebensohn et al. [13], individual grains and voids are assumed to be of a similar size, much smaller than the specimen size and the scale of variation of the applied loads. Grain orientations are characterized by rotation tensors $\mathbf{Q}^{(r)}$ $(r = 1, \ldots, N)$. All grains with a given orientation $\mathbf{Q}^{(r)}$ occupy a disconnected domain $\Omega^{(r)}$ and are collectively referred to as 'phase' r. Similarly, all voids occupy a disconnected domain $\Omega^{(0)}$ and are collectively referred to as 'phase' 0.

Grains are assumed to deform by multi-glide along K slip systems following a rigid-perfectly plastic response. In accordance with standard crystal plasticity theory, their *strength domains* are characterized in terms of a convex set:

$$P = \left\{\boldsymbol{\sigma}\colon\ |\boldsymbol{\sigma}\cdot\boldsymbol{\mu}_{(k)}| \leqslant \tau_0^{(k)},\ k = 1, \ldots, K\right\} \tag{1}$$

where $\tau_0^{(k)} > 0$ is the yield strength of the kth slip system and:

$$\boldsymbol{\mu}_{(k)} = \frac{1}{2}(\mathbf{n}_{(k)} \otimes \mathbf{m}_{(k)} + \mathbf{m}_{(k)} \otimes \mathbf{n}_{(k)}) \tag{2}$$

are second-order Schmid tensors with $\mathbf{n}_{(k)}$ and $\mathbf{m}_{(k)}$ denoting the unit vectors normal to the slip plane and along the slip direction of the kth system in a 'reference' crystal, respectively. Note that the Schmid tensors are traceless and therefore the strength domains (1) are insensitive to hydrostatic stresses. The set P is a convex polyhedron formed by the set of planes (or facets) whose equations are given by the equalities in (1). The set of vertices of P is denoted as $\hat{P}$. The strength domain $P^{(r)}$ of phase r is given by a set similarly defined in terms of rotated Schmid tensors $\boldsymbol{\mu}_{(k)}^{(r)} = \mathbf{Q}^{(r)^T}\boldsymbol{\mu}_{(k)}\mathbf{Q}^{(r)}$. The boundary $\partial P^{(r)}$ of the set $P^{(r)}$ represents the *yield surface* of phase r. The voided phase $(r = 0)$, on the other hand, cannot sustain stress. We characterize this phase as a family of 'grains' with $P^{(0)} = \{\mathbf{0}\}$.

The macroscopic plastic strength of the polycrystalline aggregate corresponds to the set of stress states that can produce macroscopic plastic flow. By homogenizing the relevant field equations, Suquet [14] and Bouchitté and Suquet [15] showed that the macroscopic plastic strength can be characterized by an *effective strength domain* defined as:

$$\widetilde{P} = \left\{\bar{\boldsymbol{\sigma}}\colon\ \exists\boldsymbol{\sigma}(\mathbf{x}) \in \mathcal{S}(\bar{\boldsymbol{\sigma}}) \text{ and } \boldsymbol{\sigma}(\mathbf{x}) \in P^{(r)} \text{ in } \Omega^{(r)},\ r = 0, \ldots, N\right\} \tag{3}$$

where $\bar{\boldsymbol{\sigma}}$ denote the macroscopic stress states, $\boldsymbol{\sigma}(\mathbf{x})$ are the underlying microscopic stress fields, and $\mathcal{S}(\bar{\boldsymbol{\sigma}})$ denotes the set of statically admissible stress fields with volume average $\bar{\boldsymbol{\sigma}}$. The effective strength domain depends on the *crystallographic* texture of the polycrystal through the set of orientations $\mathbf{Q}^{(r)}$, and on the *morphological* texture and porosity through the ensemble averages of the characteristic functions of the domains $\Omega^{(r)}$. The boundary $\partial\widetilde{P}$ of the set $\widetilde{P}$ represents the *effective yield surface* of the polycrystalline voided solid.